

AD-A050 248

NAVAL POSTGRADUATE SCHOOL MONTEREY CALIF  
A TABLE OF LANCHESTER-CLIFFORD-SCHLAEFLI FUNCTIONS. (U)  
OCT 77 J G TAYLOR, G G BROWN  
NPS55-77-39


F/6 12/1

UNCLASSIFIED

ARO-14961.1-M

NL

1 OF 1  
AD A050248

END  
DATE  
FILMED  
3-78  
DDC

AD A 050248

2

NPS55-77-39

# NAVAL POSTGRADUATE SCHOOL

Monterey, California



AD No. \_\_\_\_\_  
DDC FILE COPY

A TABLE OF  
LANCHESTER-CLIFFORD-SCHLÄFLI FUNCTIONS

by

James G. Taylor

and

Gerald G. Brown

October 1977

DISTRIBUTION STATEMENT A  
Approved for public release  
Distribution Unlimited

DDC  
RECEIVED  
FEB 22 1978  
D



NAVAL POSTGRADUATE SCHOOL  
Monterey, California

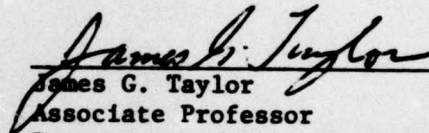
Rear Admiral Isham Linder  
Superintendent

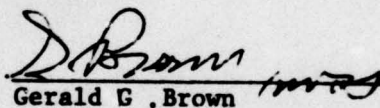
Jack R. Borsting  
Provost

This research has been partially supported by the U.S. Army Research Office, Durham, North Carolina with R&D Project No. 1L161102BH57-05 Math (funded under MIPR No. ARO 22-77) and partially by the Office of Naval Research.

Reproduction of all or part of this report is authorized.

This report was prepared by:


  
James G. Taylor  
Associate Professor  
Department of Operations Research

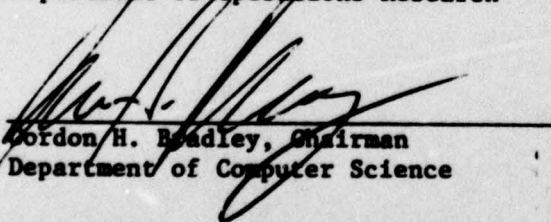
  
Gerald G. Brown  
Associate Professor  
Departments of Operations Research  
and Computer Science

Reviewed by:

Released by:

  
Michael G. Sovereign, Chairman  
Department of Operations Research

  
for Robert R. Fossum  
Dean of Research

  
Gordon H. Roadley, Chairman  
Department of Computer Science

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER <b>(14)</b> NPS55-77-39	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) <b>(9)</b> A Table of Lanchester-Clifford-Schläfli Functions	5. TYPE OF REPORT & PERIOD COVERED <b>(9)</b> Technical rept.	
7. AUTHOR(s) <b>(10)</b> James G. Taylor <del>and</del> Gerald G. Brown	6. PERFORMING ORG. REPORT NUMBER  TRACT OR GRANT NUMBER(s) <b>(11)</b> 4801411, 11, 161102BH57 <b>(12)</b> 4801411, 05	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, Ca. 93940	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS ARO 22-77 & 61153N, NR-277-229, RR 014-11-01 N 0001477 YR70192	
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Durham, North Carolina	<b>(11)</b> October 1977	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) <b>(18)</b> ARO / <b>(19)</b> 14961.1-M	13. NUMBER OF PAGES 90 <b>(12)</b> 92p.	
15. SECURITY CLASS. (of this report) Unclassified		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Lanchester Theory of Combat      Special Functions Combat Modelling      Deterministic Combat Attrition Attrition Modelling      Lanchester-Clifford-Schläfli Functions Combat Dynamics		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report contains the most extensive set of tables currently available of Lanchester-Clifford-Schläfli (LCS) functions. These functions may be used to analyze Lanchester-type combat between two homogeneous forces modelled by power attrition-rate coefficients with "no offset." Theoretical background for the LCS functions is given, as well as a narrative description of the physical circumstances under which the associated Lanchester-type combat model may be expected to be applicable. Numerical examples are given		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE  
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

251450

next  
page

pk



UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20. Cont.

to illustrate the use of the LCS functions for analyzing "aimed-fire" combat modelled by the power attrition-rate coefficients with "no offset." Our results and these tabulations allow one to study this particular variable-coefficient combat model almost as easily and thoroughly as Lanchester's classic constant-coefficient model.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)



# A TABLE OF LANCHESTER-CLIFFORD-SCHLÄFLI FUNCTIONS

by

James G. Taylor  
Department of Operations Research

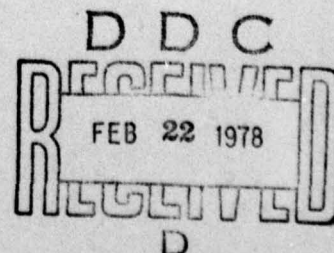
and

Gerald G. Brown  
Departments of Operations Research  
and Computer Science

ACCESSION for	
RTIS	White Section <input checked="" type="checkbox"/>
DDC	Buff Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	AVAIL. and/or SPECIAL
A	

Naval Postgraduate School  
Monterey, California

This research was partially supported by the U. S. Army Research Office, Durham, R&D Project No. 1L161102BH57-05 Math (funded under MIPR No. ARO 22-77) and partially by the Office of Naval Research.



## TABLE OF CONTENTS

	PAGE
1. Introduction .....	1
2. Variable-Coefficient Lanchester-Type Equations of Modern Warfare .....	2
3. Combat Modelled with Power Attrition-Rate Coefficients.....	5
4. Lanchester-Clifford-Schlafli (LCS) Functions .....	12
5. Use of LCS Functions for Analyzing Combat .....	18
6. Tabulation of LCS Functions .....	21
7. Numerical Examples .....	22
8. Final Remarks .....	32
References .....	35
Appendix: Tabulation of LCS Functions $F_{\alpha}(x)$ , $H_{1-\alpha}(x)$ , and $T_{\alpha}(x)$ for 25 Fractional Values of $\alpha$ .....	36



## 1. Introduction

Lanchester-type\* differential-equation combat models are an important tool for analyzing many important problems of military operations research. In such a combat model, a so-called attrition-rate coefficient represents the fire effectiveness of a particular weapon-system type against a particular target type, i.e. the weapon-system type's effective firepower against such a target. Time-dependent attrition-rate coefficients are used to model temporal variations in firepower on the battlefield. Thus, we see that time-dependent attrition-rate coefficients are important (and, in fact, essential [4-6]) for the quantitative analysis of hypothetical combat.

Militarily realistic computer-based Lanchester-type models of quite complex military systems have been developed for almost the entire spectrum of combat operations, from combat between battalion-sized units to theater-level operations. Nevertheless, a simple combat model may yield a clearer understanding of significant interrelationships that are difficult to perceive in a more complex model, and such insights can subsequently provide valuable guidance for more detailed computerized investigations. In this report we consider such a simplified variable-coefficient Lanchester-type model of combat between two homogeneous forces.

For this variable-coefficient Lanchester-type model of combat between two homogeneous forces, different functional forms for the attrition-rate coefficients lead to different mathematical functions being involved in representing and computing the force-level trajectories. In a previous paper [5] we have discussed the plausibility of the hypothesis that except for the special case of a constant ratio of attrition-rate coefficients,

---

\* So-called after pioneering work of F. W. Lanchester [3].



the solutions to such differential equations cannot be represented in terms of "elementary" functions of analysis. Thus, new transcendental functions arise in the study of combat modelled with time-dependent attrition-rate coefficients. In particular, we have previously introduced [5-6] so-called Lanchester-Clifford-Schläfli (LCS) functions for analyzing combat modelled with power attrition-rate coefficients with "no offset" (see Section 3 below).

In the Appendix to this report is contained the most extensive set of tables currently available for the LCS functions: it contains tables of five-decimal-place values of the hyperbolic-like LCS functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and  $T_{\alpha}(x)$  (see Section 4 below) for 25 fractional values of  $\alpha$  (see Section 6 below). The main body of this report provides the theoretical and modelling background for the use of these tables. In particular, we examine a model of a constant-speed attack on a static defensive position and show how associated range-dependent kill rates give rise to time-dependent attrition-rate coefficients with "no offset." Numerical computations are presented to illustrate the use of the LCS functions for analyzing such "aimed-fire" combat. As a consequence of the availability of these tables, one can now study this variable-coefficient combat model almost as easily and thoroughly as Lanchester's classic constant-coefficient model.

## 2. Variable-Coefficient Lanchester-Type Equations of Modern Warfare.

We consider combat between two homogeneous forces modelled by the following variable-coefficient Lanchester-type [3] (see [4,5]) equations of modern warfare

$$\begin{cases} \frac{dx}{dt} = -a(t)y & \text{with } x(0) = x_0, \\ \frac{dy}{dt} = -b(t)x & \text{with } y(0) = y_0, \end{cases} \quad (2.1)$$

where  $t = 0$  denotes the time at which the battle begins,  $x(t)$  and  $y(t)$  denote the numbers of  $X$  and  $Y$  at time  $t$ , and  $a(t)$  and  $b(t)$  denote time-dependent Lanchester attrition-rate coefficients, which represent the effectiveness of each side's fire. These coefficients depend on variables such as force separation, tactical posture of targets, rate of target acquisition, firing rate, etc. (see [4-7] for further details). Variable attrition-rate coefficients are used to model temporal variations in firepower on the battlefield. In any analysis of combat, moreover, we should use the above equations (2.1) only for  $x$  and  $y \geq 0$  and, for example, set  $dx/dt = 0$  when  $x = 0$ , since negative force levels have no physical meaning.

Mathematically, we assume that the attrition-rate coefficients  $a(t)$  and  $b(t)$  are defined, positive, and continuous for  $t_0 < t < +\infty$  with  $t_0 \leq 0$ . We also assume that  $a(t)$  and  $b(t) \in L(t_0, T)$  for any finite  $T \geq t_0$ . We further take  $a(t)$  and  $b(t)$  to be given in the form

$$a(t) = k_a g(t), \quad \text{and} \quad b(t) = k_b h(t), \quad (2.2)$$

where  $k_a$  and  $k_b$  are positive constants chosen so that  $a(t)/b(t) = k_a/k_b$  when  $g(t) \equiv h(t)$ . We introduce the combat-intensity parameter  $\lambda_I$  and the relative-fire-effectiveness parameter  $\lambda_R$  defined by



$$\lambda_I = \sqrt{k_a k_b}, \quad \text{and} \quad \lambda_R = k_a/k_b. \quad (2.3)$$

From our assumptions about  $a(t)$  and  $b(t)$ , it follows that, for example,  $a(t) \notin L(t_0, T)$  implies  $\int_{t_0}^T a(t) dt = +\infty$ .

The  $X$  force level as a function of time may be represented as [5,6]

$$x(t) = x_0 \{C_Y(0)C_X(t) - S_Y(0)S_X(t)\} - y_0 \sqrt{\lambda_R} \{C_X(0)S_X(t) - S_X(0)C_X(t)\}, \quad (2.4)$$

where the hyperbolic-like general Lanchester functions (GLF)  $C_X(t)$  and  $S_X(t)$  are linearly-independent solutions to the  $X$  force-level equation

$$\frac{d^2 x}{dt^2} - \left\{ \frac{1}{a(t)} \frac{da}{dt} \right\} \frac{dx}{dt} - a(t)b(t)x = 0, \quad (2.5)$$

with initial conditions

$$\begin{aligned} C_X(t_0) &= 1, & S_X(t_0) &= 0, \\ \{1/a(t_0)\} dC_X/dt(t_0) &= 0, & \{1/a(t_0)\} dS_X/dt(t_0) &= 1/\sqrt{\lambda_R}. \end{aligned} \quad (2.6)$$

Here  $t_0$  denotes the largest finite time at which  $a(t)$  or  $b(t)$  ceases to be defined, positive, or continuous. The  $Y$  force level as a function of time is given by a similar expression, with  $C_Y(t)$  and  $S_Y(t)$  being analogously defined for the corresponding  $Y$  force-level equation.

It is sometimes convenient to introduce the new independent variable  $\tau$  defined by



$$\tau = \int_{t_0}^t \sqrt{a(s)b(s)} \, ds . \quad (2.7)$$

It is readily seen that the transformation  $\tau = \tau(t)$  is well defined and invertible. Let us denote  $\tau(0)$  as  $\tau_0$ . We observe that  $t_0 \leq 0$  implies that  $\tau_0 \geq 0$ . If we denote the "average intensity of combat" as  $\sqrt{a(t)b(t)}$ , then

$$\sqrt{a(t)b(t)} \, t = \left\{ \left( \frac{1}{t} \right) \int_0^t \sqrt{a(s)b(s)} \, ds \right\} t = \tau - \tau_0 . \quad (2.8)$$

The substitution (2.7) transforms (2.5) into

$$\frac{d^2 x}{d\tau^2} - \left( \frac{1}{2} \right) \left\{ \frac{d}{d\tau} \ln R(\tau) \right\} \frac{dx}{d\tau} - x = 0 , \quad (2.9)$$

with initial conditions

$$x(\tau_0) = x_0 , \quad \text{and} \quad \{1/\sqrt{R(\tau_0)}\} \, dx/d\tau(\tau_0) = -y_0 ,$$

where  $R(\tau) = a(t)/b(t)$ .

### 3. Combat Modelled with Power Attrition-Rate Coefficients.

The above equations (2.1) basically apply to "aimed-fire" combat when target-acquisition times do not depend on the numbers of targets available (see [5,6] for further details). A large class of tactical situations of interest can be modelled with the following general power attrition-rate coefficients [5-7]

$$a(t) = k_a (t + C)^\mu, \quad \text{and} \quad b(t) = k_b (t + C + A)^\nu, \quad (3.1)$$

where  $A$  and  $C \geq 0$ . We will call  $A$  the offset parameter, since it allows us to model (with  $\mu$  and  $\nu \geq 0$ ) battles between opposing weapon systems with different maximum effective ranges (see [5,6]). We will call  $C$  the starting parameter, since it allows us to model (again, with  $\mu$  and  $\nu \geq 0$ ) battles that begin within the maximum effective ranges of the two opposing systems. We observe that for the general power attrition-rate coefficients (3.1) we have  $t_0 = -C$ , and  $\mu$  and  $\nu$  must be  $> -1$  in order that  $a(t)$  and  $b(t) \in L(t_0, T)$ .

The above nomenclature is motivated and possible applications of our work are indicated by considering S. Bonder's model of the constant-speed attack on a static defensive position (see [4-7] for further details)

$$\frac{dx}{dt} = -\alpha(r)y, \quad \text{and} \quad \frac{dy}{dt} = -\beta(r)x, \quad (3.2)$$

where  $r$  denotes the range between opposing forces, and  $\alpha(r)$  and  $\beta(r)$  denote range-dependent attrition-rate coefficients. Range is related to time by

$$r(t) = R_0 - vt, \quad (3.3)$$

where  $R_0$  denotes the opening range of battle and  $v > 0$  denotes the constant attack speed. For example, let us consider the constant-speed attack of a homogeneous  $Y$  force against the static defensive position of a homogeneous  $X$  force. Figure 1 diagrammatically portrays this situation.

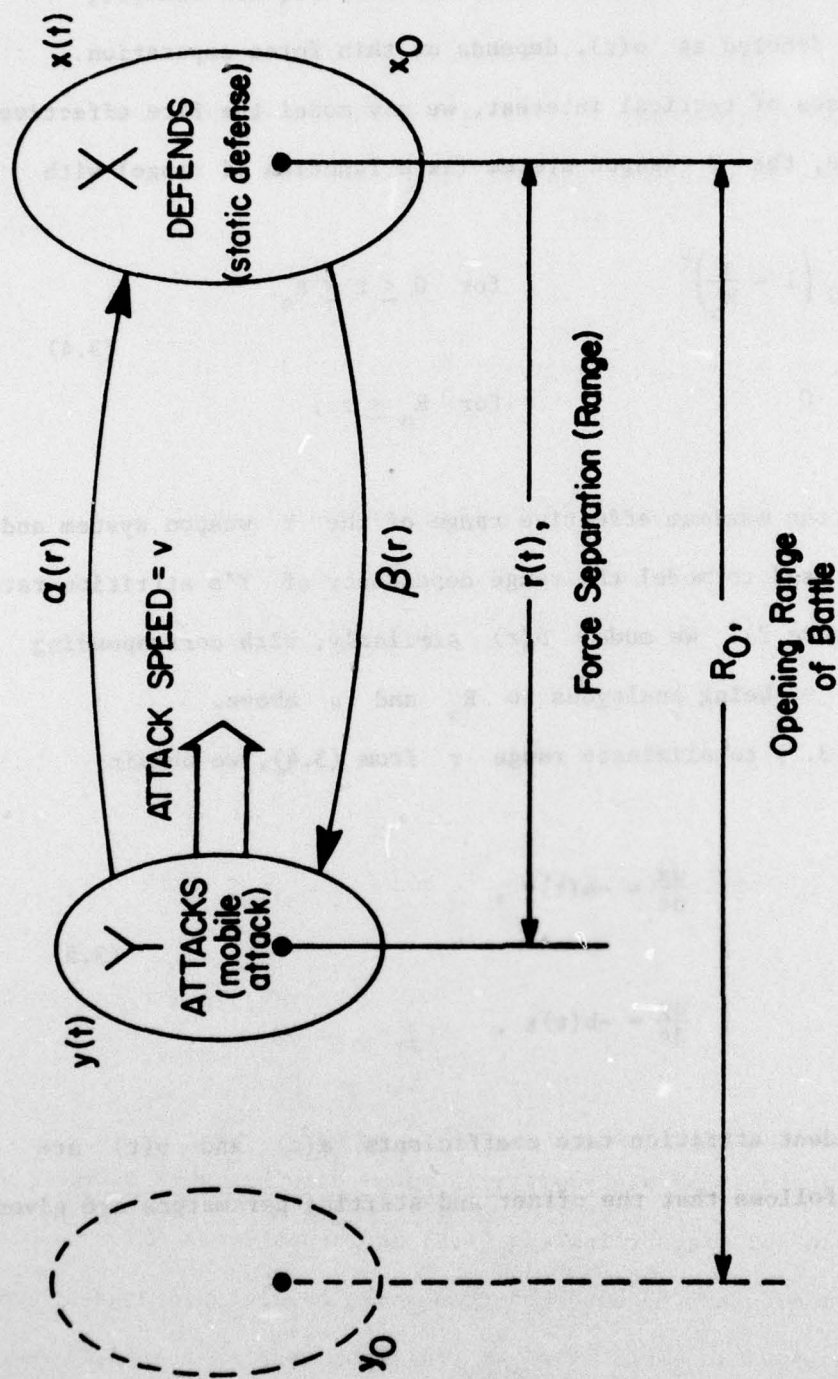


Figure 1. Diagram of Bonder's constant-speed attack model.

Force separation,  $r(t)$ , is given by  $r(t) = R_0 - vt$ .



The basic idea is that force separation, i.e. range between the opposing forces, changes over time, and the fire effectiveness of, for example, a single Y firer, denoted as  $\alpha(r)$ , depends on this force separation.

In many cases of tactical interest, we may model the fire effectiveness of, for example, the Y weapon system (as a function of range) with

$$\alpha(r) = \begin{cases} \alpha_0 \left(1 - \frac{r}{R_\alpha}\right)^\mu & \text{for } 0 \leq r \leq R_\alpha, \\ 0 & \text{for } R_\alpha \leq r, \end{cases} \quad (3.4)$$

where  $R_\alpha$  denotes the maximum effective range of the Y weapon system and  $\mu \geq 0$ . Here  $\mu$  is used to model the range dependency of Y's attrition-rate coefficient (see Figure 2). We model  $\beta(r)$  similarly, with corresponding quantities  $R_\beta$  and  $\nu$  being analogous to  $R_\alpha$  and  $\mu$  above.

If we use (3.3) to eliminate range  $r$  from (3.4), we obtain

$$\begin{cases} \frac{dx}{dt} = -a(t)y, \\ \frac{dy}{dt} = -b(t)x, \end{cases} \quad (3.5)$$

where the time-dependent attrition-rate coefficients  $a(t)$  and  $b(t)$  are given by (3.1). It follows that the offset and starting parameters are given by

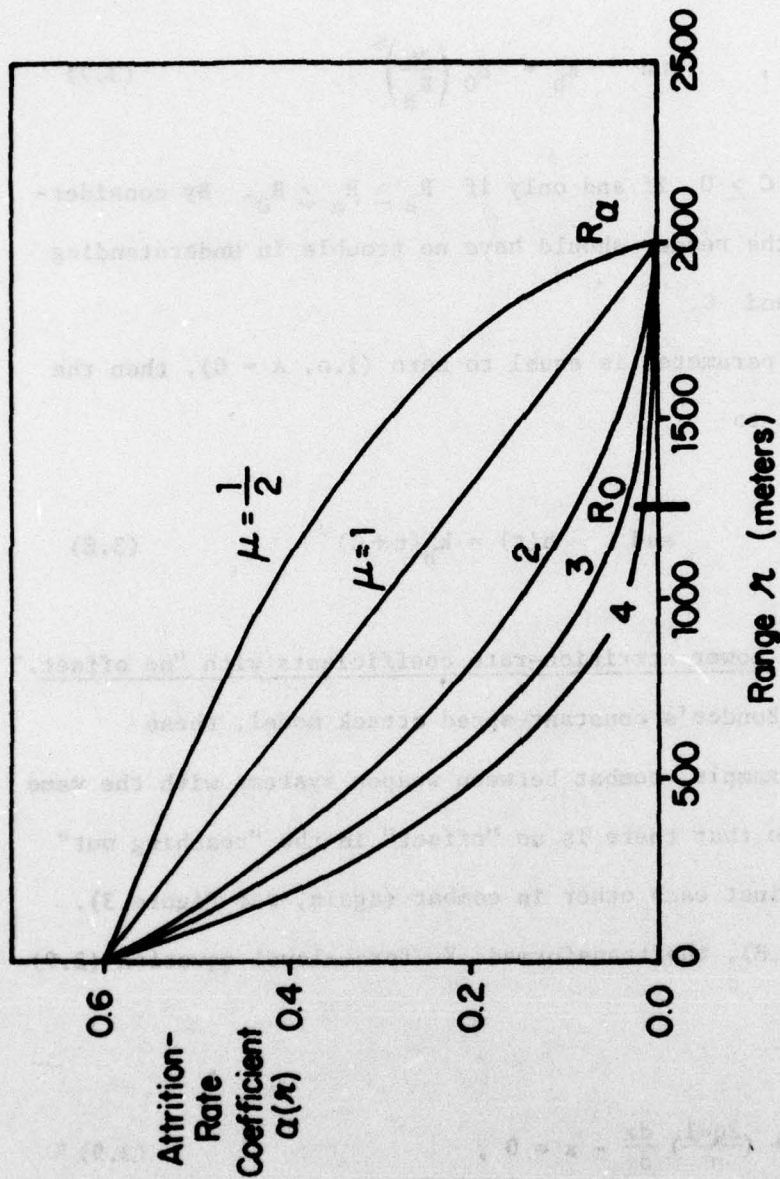


Figure 2. Dependence of Y's attrition-rate coefficient  $\alpha(r)$  on the exponent  $\mu$  with the maximum effective range of the weapon system and kill rate at zero range held constant. [NOTES: 1. The maximum effective range of the system is denoted as  $R_\alpha = 2000$  meters. 2.  $\alpha(0) = \alpha_0 = 0.6X$  casualties/(unit time  $\times$  number of Y firers) denotes the weapon-system kill rate for Y at zero force separation (range). 3. The opening range of battle is denoted as  $R_0 = 1250$  meters and (as shown)  $R_0 < R_\alpha$ .]

$$A = \left( \frac{R_\beta - R_\alpha}{v} \right), \quad \text{and} \quad C = \left( \frac{R_\alpha - R_0}{v} \right), \quad (3.6)$$

and that

$$k_a = \alpha_0 \left( \frac{v}{R_\alpha} \right)^\mu, \quad \text{and} \quad k_b = \beta_0 \left( \frac{v}{R_\beta} \right)^\nu. \quad (3.7)$$

We observe that  $A$  and  $C \geq 0$  if and only if  $R_\beta \geq R_\alpha \geq R_0$ . By considering (3.6) and Figure 3, the reader should have no trouble in understanding our terminology for  $A$  and  $C$ .

When the offset parameter is equal to zero (i.e.  $A = 0$ ), then the coefficients (3.1) reduce to

$$a(t) = k_a (t+C)^\mu, \quad \text{and} \quad b(t) = k_b (t+C)^\nu. \quad (3.8)$$

We will refer to (3.8) as power attrition-rate coefficients with "no offset."

As we have seen above in Bonder's constant-speed attack model, these coefficients model, for example, combat between weapon systems with the same maximum effective range so that there is no "offset" in the "reaching out" of the weapon systems against each other in combat (again, see Figure 3). For these coefficients (3.8), the transformed  $X$  force-level equation (2.9) becomes

$$\frac{d^2 x}{d\tau^2} + \left( \frac{2q-1}{\tau} \right) \frac{dx}{d\tau} - x = 0, \quad (3.9)$$

with initial conditions



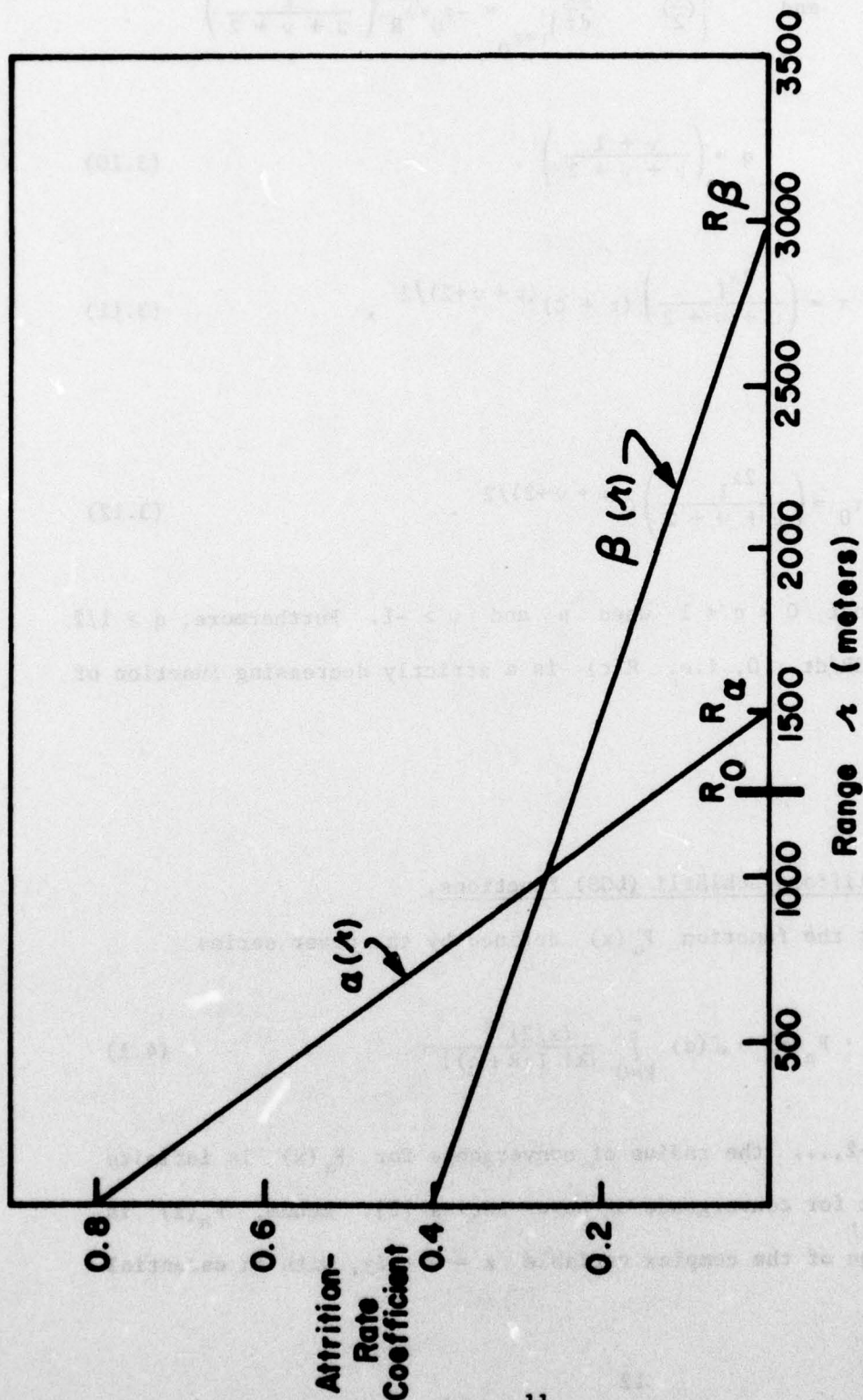


Figure 3. Explanation of the offset parameter  $A$  and the starting parameter  $C$  for power attrition-rate coefficients modelling constant-speed attack. [NOTES: 1. The maximum effective ranges of the  $X$  and  $Y$  weapon systems are denoted as  $R_\alpha$  and  $R_\beta$ , respectively. 2. The opening range of battle is denoted as  $R_0$  and (as shown)  $R_0 < \min(R_\alpha, R_\beta)$ . 3. The offset parameter is given by  $A = (R_\beta - R_\alpha)/v$ . 4. The starting parameter is given by  $C = (R_\alpha - R_0)/v$ .]

$$x(\tau_0) = x_0, \quad \text{and} \quad \left\{ \left( \frac{\tau}{2} \right)^{2q-1} \frac{dx}{d\tau} \right\}_{\tau=\tau_0} = -y_0 \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1}.$$

Here

$$q = \left( \frac{\nu + 1}{\mu + \nu + 2} \right), \quad (3.10)$$

$$\tau = \left( \frac{2\lambda_I}{\mu + \nu + 2} \right) (t + C)^{(\mu + \nu + 2)/2}, \quad (3.11)$$

and

$$\tau_0 = \left( \frac{2\lambda_I}{\mu + \nu + 2} \right) C^{(\mu + \nu + 2)/2}. \quad (3.12)$$

Let us observe that  $0 < q < 1$  when  $\mu$  and  $\nu > -1$ . Furthermore,  $q > 1/2$  if and only if  $dR/dt < 0$ , i.e.  $R(t)$  is a strictly decreasing function of time.

#### 4. Lanchester-Clifford-Schläfli (LCS) Functions.

Consider the function  $F_\alpha(x)$  defined by the power series

$$F_\alpha(x) = \Gamma(\alpha) \sum_{k=0}^{\infty} \frac{(x/2)^{2k}}{\{k! \Gamma(k+\alpha)\}}. \quad (4.1)$$

For  $\alpha \neq 0, -1, -2, \dots$  the radius of convergence for  $F_\alpha(x)$  is infinite by the ratio test for convergence of power series [2]. Hence,  $F_\alpha(z)$  is an entire function of the complex variable  $z = x + iy$ , with an essential

singularity at the point at infinity. Now consider the function  $H_\alpha(x)$  defined by the infinite series

$$H_\alpha(x) = \Gamma(\alpha) \sum_{k=0}^{\infty} \frac{(x/2)^{2(k+\alpha)}}{\{k! \Gamma(k+\alpha+1)\}}. \quad (4.2)$$

Observing that

$$H_\alpha(x) = (1/\alpha)(x/2)^{2\alpha} F_{\alpha+1}(x), \quad (4.3)$$

we see that for  $\alpha > 0$  the infinite series (4.2) is uniformly convergent on compact subsets of the complex plane. From (4.3) one can readily deduce the recursive relation

$$F_\alpha(x) = F_{\alpha+1}(x) + \left\{ \frac{(x/2)^2}{\alpha(\alpha+1)} \right\} F_{\alpha+2}(x). \quad (4.4)$$

We will call the functions  $F_\alpha(x)$  and  $H_\alpha(x)$  Lanchester-Clifford-Schlöfli (LCS) functions (see Note 10 on pp. 66-67 of [5]). Other properties are readily deduced and are given in Table I.

The function  $F_\alpha(x)$  satisfies the linear second-order ordinary differential equation

$$\frac{d^2 F_\alpha}{dx^2} + \left( \frac{2\alpha-1}{x} \right) \frac{dF_\alpha}{dx} - F_\alpha = 0, \quad (4.5)$$

with initial conditions



Table I. Properties of the LCS Functions  $F_{\alpha}(x)$  and  $H_{\alpha}(x)$ .

1.  $dF_{\alpha}/dx = (x/2)^{1-2\alpha} H_{\alpha}(x)$
2.  $dH_{\alpha}/dx = (x/2)^{2\alpha-1} F_{\alpha}(x)$
3.  $F_{\alpha}(x)F_{1-\alpha}(x) - H_{\alpha}(x)H_{1-\alpha}(x) = 1 \quad \forall x$   
where  $\alpha$  is not an integer (including zero)
4.  $F_{\alpha}(x=0) = 1$
5.  $H_{\alpha}(x=0) = 0 \quad \text{for } \alpha > 0$
6.  $dF_{\alpha}/dx(x=0) = 0$
7.  $\{(x/2)^{1-2\alpha} dH_{\alpha}/dx\}_{x=0} = 1$
8.  $F_{1/2}(x) = \cosh x$
9.  $H_{1/2}(x) = \sinh x$

$$F_{\alpha}(0) = 1, \quad \text{and} \quad \frac{dF_{\alpha}}{dx}(0) = 0,$$

while  $H_{\alpha}(x)$  satisfies

$$\frac{d^2 H_{\alpha}}{dx^2} - \left(\frac{2\alpha-1}{x}\right) \frac{dH_{\alpha}}{dx} - H_{\alpha} = 0, \quad (4.6)$$

with initial conditions

$$H_{\alpha}(0) = 0, \quad \text{and} \quad \left\{ \left(\frac{x}{2}\right)^{1-2\alpha} \frac{dH_{\alpha}}{dx} \right\}_{x=0} = 1.$$

Thus,  $\{F_{\alpha}, H_{1-\alpha}\}$  is a fundamental system of solutions to

$$\frac{d^2 F}{dx^2} + \left(\frac{2\alpha-1}{x}\right) \frac{dF}{dx} - F = 0, \quad (4.7)$$

with Wronskian  $W(F_{\alpha}, H_{1-\alpha}) = (x/2)^{1-2\alpha}$ . It follows that the GLF for the X and Y force-level equations for combat modelled with the attrition-rate coefficients (3.8) are given by

$$C_X(t) = F_q(\tau(t)), \quad S_X(t) = \left(\frac{\lambda_I}{\mu + \nu + 2}\right)^{2q-1} H_p(\tau(t)), \quad (4.8)$$

$$C_Y(t) = F_p(\tau(t)), \quad S_Y(t) = \left(\frac{\lambda_I}{\mu + \nu + 2}\right)^{1-2q} H_q(\tau(t)), \quad (4.9)$$

where  $p = 1-q$ . If we define

$$T_{\alpha}(x) = H_{1-\alpha}(x)/F_{\alpha}(x), \quad (4.10)$$

then

$$T_X(t) = \frac{S_X(t)}{C_X(t)} = \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1} \frac{H_p(\tau(t))}{F_q(\tau(t))}, \quad (4.11)$$

or

$$T_X(t) = \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1} T_q(\tau(t)), \quad (4.12)$$

where  $T_X(t)$  denotes a hyperbolic-like GLF, which corresponds to the hyperbolic tangent. Observing that for  $\mu, \nu > -1$ ,  $\lim_{t \rightarrow +\infty} \tau(t) = +\infty$ , we see that  $T_{\alpha}(x)$  is a strictly increasing function of  $x$  on the interval  $[0, +\infty)$  and

$$0 \leq T_{\alpha}(x) < \frac{\Gamma(1-\alpha)}{\Gamma(\alpha)} \quad \text{for } 0 \leq x < +\infty, \quad (4.13)$$

with

$$\lim_{x \rightarrow +\infty} T_{\alpha}(x) = \frac{\Gamma(1-\alpha)}{\Gamma(\alpha)}, \quad (4.14)$$

since by the results of Taylor and Comstock [7] the parity-condition parameter  $Q^* = Q^*(\mu, \nu, C = 0)$  is given by

$$\lim_{t \rightarrow +\infty} T_X(t) = \frac{1}{Q^*(\mu, \nu, 0)} = \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1} \frac{\Gamma(p)}{\Gamma(q)}. \quad (4.15)$$

We recall that Taylor and Comstock [7] have introduced the so-called parity-condition parameter  $Q^*$  as the value (or range of such values) for the initial condition  $Q$  to the initial-value problem



$$\begin{cases} \frac{dE_X^-}{dt} = -\frac{1}{\sqrt{\lambda_R}} a(t) E_Y^- & \text{with } E_X^-(t_0) = 1, \\ \frac{dE_Y^-}{dt} = -\sqrt{\lambda_R} b(t) E_X^- & \text{with } E_Y^-(t_0) = Q, \end{cases} \quad (4.16)$$

such that  $E_X^-(t; Q^*)$  and  $E_Y^-(t; Q^*) > 0$  for all  $t \geq t_0$ . In other words,  $Q^*$  is the value of  $Q$  in (4.16) above such that neither  $E_X^-$  nor  $E_Y^-$  ever become zero. In this case, both  $E_X^-(t; Q^*)$  and  $E_Y^-(t; Q^*)$  are positive, strictly decreasing functions, similar to decreasing exponentials. Thus, we may call  $Q^*$  "the Y equivalent of an X force of unit strength," since the forces are "at parity," with neither force being annihilated in finite time. Taylor and Comstock have shown that for either  $a(t) \notin L(0, +\infty)$  or  $b(t) \notin L(0, +\infty)$ , then  $Q^*$  is unique and given by

$$\lim_{t \rightarrow +\infty} \frac{S_X(t)}{C_X(t)} = \frac{1}{Q^*}. \quad (4.17)$$

The significance of the parity-condition parameter  $Q^*$  is that it allows us to predict force annihilation as the following theorem shows.

**THEOREM 1** (Taylor and Comstock [7]): Assume that either  $a(t) \notin L(0, +\infty)$  or  $b(t) \notin L(0, +\infty)$ . Then the X force will be annihilated in finite time if and only if

$$\frac{x_0}{y_0} < \sqrt{\lambda_R} \left\{ \frac{C_X(0) - Q^* S_X(0)}{Q^* C_Y(0) - S_Y(0)} \right\}. \quad (4.18)$$

## 5. Use of LCS Functions for Analyzing Combat.

The Lanchester-Clifford-Schläfli (LCS) functions  $F_\alpha(x)$  and  $H_\alpha(x)$  are useful for analyzing "aimed-fire" combat (see Section 3 above) modelled with the power attrition-rate coefficients with "no offset" (3.8), which we rewrite here as

$$a(t) = k_a(t + C)^\mu, \quad \text{and} \quad b(t) = k_b(t + C)^\nu. \quad (5.1)$$

In other words, the LCS functions arise in solving the differential combat model (2.1) with attrition-rate coefficients (5.1). In order that both  $a(t)$  and  $b(t) \in L(t_0, T)$ , we must have  $\mu$  and  $\nu > -1$ . Military situations modelled by these equations have been discussed in Section 3 above, e.g. combat between two weapon systems with the same maximum effective range. For such combat, the LCS functions may be used to

- (1) compute force-level declines,
  - (2) predict force annihilation,
- and
- (3) predict the time of force annihilation.

Let us now see how the LCS functions may be used to obtain the above information about force-level declines and force-annihilation prediction. According to (2.4), (4.8), and (4.9) above, the  $X$  force level is given by

$$x(t) = x_0 \{ F_p(\tau_0) F_q(\tau(t)) - H_q(\tau_0) H_p(\tau(t)) \} - y_0 \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1} \{ F_q(\tau_0) H_p(\tau(t)) - H_p(\tau_0) F_q(\tau(t)) \}, \quad (5.2)$$

where  $q$  is given by (3.10),  $p = 1-q$ , and  $\tau(t)$  is given by (3.11), which we rewrite as

$$\tau(t) = \left( \frac{2\lambda_I}{\mu + \nu + 2} \right) (t + C)^{(\mu+\nu+2)/2}. \quad (5.3)$$

The time to annihilate the  $X$  force\* is determined by  $x(t_a^X) = 0$ , and it follows that

$$T_q(\tau(t_a^X)) = \frac{x_0 F_p(\tau_0) + y_0 \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} H_p(\tau_0)}{x_0 H_q(\tau_0) + y_0 \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} F_q(\tau_0)}, \quad (5.4)$$

where from (4.10)

$$T_q(\tau(t)) = H_p(\tau(t))/F_q(\tau(t)), \quad (5.5)$$

and we recall that  $p + q = 1$ . It follows that the time to annihilate  $X$ ,  $t_a^X$ , is given by

---

\* If we multiply the first equation of (2.1) by  $y$ , the second by  $x$ , add, and integrate, we obtain

$$x(t) y(t) = x_0 y_0 - \int_0^t \{a(s) y^2(s) + b(s) x^2(s)\} ds,$$

which shows that  $x(t)$  and  $y(t)$  can have at most one finite zero. Hence, if  $x(t_a^X) = 0$ , then we know that  $y(t) > 0$  for all  $t \geq 0$ .



$$t_a^X = \tau^{-1} \left\{ T_q^{-1} \left[ \frac{x_0 F_p(\tau_0) + y_0 \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} H_p(\tau_0)}{x_0 H_q(\tau_0) + y_0 \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} F_q(\tau_0)} \right] \right\}. \quad (5.6)$$

Taylor and Comstock [7] have shown that  $T_q(\tau)$  is strictly increasing and satisfies (see also (4.12) above)

$$0 \leq T_q(\tau) < \Gamma(p)/\Gamma(q), \quad (5.7)$$

where  $p = 1-q$ . It follows that in order for  $X$  to be annihilated in finite time, the right-hand side of (5.4) must be less than  $\Gamma(p)/\Gamma(q)$ . Let us observe that for  $t_0 = -C = 0$ , (5.4) simplifies to

$$T_q(\tau(t_a^X)) = \frac{x_0}{y_0 \sqrt{\lambda_R}} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{p-q}. \quad (5.8)$$

Thus, we have proved the following theorem concerning force-annihilation prediction.

**THEOREM 2:** Consider combat between two homogeneous forces modelled by (2.1) with power attrition-rate coefficients (5.1). Assume that  $\mu$  and  $\nu > -1$  and that the above equations hold for all time. Then the  $X$  force will be annihilated in finite time if and only if

$$\Gamma(q) \left\{ x_0 F_p(\tau_0) + y_0 \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} H_p(\tau_0) \right\} \\ < \Gamma(p) \left\{ x_0 H_q(\tau_0) + y_0 \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} F_q(\tau_0) \right\}, \quad (5.9)$$

where  $q = (\nu + 1)/(\mu + \nu + 2)$  and  $p = 1 - q$ . For  $\tau_0 = 0$  (i.e.  $C = 0$  so that  $\tau_0 = 0$ ),  $X$  will be annihilated in finite time if and only if

$$\frac{x_0}{y_0} < \frac{\Gamma(p)}{\Gamma(q)} \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p}. \quad (5.10)$$

## 6. Tabulation of LCS Functions.

This report contains the most extensive set of tables of the Lanchester-Clifford-Schläfli functions currently available. The Appendix contains tables of five-decimal-place values of the hyperbolic-like LCS functions  $F_\alpha(x)$ ,  $H_{1-\alpha}(x)$ , and  $T_\alpha(x)$  for various values of the argument  $x$ , namely  $x = 0.00$  (0.01) 2.00 (0.1) 10.0, and  $\alpha = 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 2/7, 3/7, 4/7, 5/7, 4/9, 5/9, 3/11, 5/11, 6/11, 8/11, 5/13, 8/13, 5/17, 12/17, 5/21, \text{ and } 16/21$ . These values of the index  $\alpha$  correspond to  $\mu, \nu = 0, 1/4, 1/2, 1, 1\frac{1}{2}, 2, \text{ and } 3$  in (3.8) and allow one to analyze, for example, a fairly wide variety of range capabilities for weapon systems in the constant-speed-attack model of Section 3. These

tables have been calculated by the recursive means given in Section 8 of [5]. A less extensive tabulation (namely, for  $\alpha = 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 3/7$ , and  $4/7$  corresponding to  $\mu, \nu = 0, 1, 2, 3$ ) is to be found in a companion report [8].

A representative tabulation of the hyperbolic-like LCS functions  $F_\alpha(x)$ ,  $H_{1-\alpha}(x)$ , and  $T_\alpha(x)$  is given in, for example, Tables 8A and 8B of the Appendix for  $\alpha = 3/5$ . The values of the argument  $x$  are the same as those used for the tabulation of the hyperbolic functions by Abramowitz and Stegun [1]. We observe from Table 8B and (4.13) that the limiting value of  $T_\alpha(x)$  as  $x \rightarrow +\infty$  (here  $\alpha = 3/5$ ) is quickly reached, with three-decimal-place accuracy already attained for  $x = 4.5$ . Moreover, the use of these tables (specifically, Tables 8A and 8B of the Appendix) for combat analysis is illustrated in the next section.

## 7. Numerical Examples

In this section we examine a couple of numerical examples to show some of the insights that may be gained into the dynamics of combat between two homogeneous forces from our results (see also [6]). These examples illustrate the use of the LCS functions  $F_\alpha(x)$ ,  $H_{1-\alpha}(x)$ , and  $T_\alpha(x)$  for analyzing "aimed-fire" combat modelled with the power attrition-rate coefficients with "no offset" (5.1). As in [4-7], we consider S. Bonder's model (3.2) for the constant-speed attack against a static defensive position. We will focus on the use of the LCS functions for predicting force annihilation, since the computing of force-level trajectories with Lanchester functions is adequately handled elsewhere (see [4-5]).



Let us accordingly consider the constant-speed attack of a homogeneous Y force against the static defensive position of a homogeneous X force (see Section 3 above for further modelling details, especially Figure 1). For our numerical computations, we assume that the fire effectiveness of the Y weapon system varies linearly with range, i.e.

$$\alpha(r) = \begin{cases} \alpha_0 \left(1 - \frac{r}{R_\alpha}\right) & \text{for } 0 \leq r \leq R_\alpha, \\ 0 & \text{for } R_\alpha \leq r, \end{cases} \quad (7.1)$$

and that the fire effectiveness of the X weapon system varies quadratically with range, i.e.

$$\beta(r) = \begin{cases} \beta_0 \left(1 - \frac{r}{R_\beta}\right)^2 & \text{for } 0 \leq r \leq R_\beta, \\ 0 & \text{for } R_\beta \leq r, \end{cases} \quad (7.2)$$

with  $R_\alpha = R_\beta$ , i.e. both weapon systems have the same maximum effective range. In other words,  $\mu = 1$  in (3.4) and  $\nu = 2$  for  $\beta(r)$ . We consider a battle modelled by the input data given in Table II. In terms of time as the independent variable, the attrition-rate coefficients (7.1) and (7.2) become via (3.3)

$$a(t) = k_a(t + C) \quad \text{and} \quad b(t) = k_b(t + C)^2, \quad (7.3)$$

Table II. Input Data for Numerical Examples

$$\mu = 1, \quad \nu = 2$$

$$\alpha_0 = 0.06 \text{ X casualties/minute/Y firer}$$

$$\beta_0 = 0.6 \text{ Y casualties/minute/X firer}$$

$$R_\alpha = R_\beta = 2000 \text{ meters}$$

$$v = 5 \text{ miles/hour}$$

where  $R_\alpha = R_\beta$ ,

$$C = \frac{R_\alpha - R_0}{v}, \quad k_a = \frac{\alpha_0 v}{R_\alpha}, \quad \text{and} \quad k_b = \beta_0 \left( \frac{v}{R_\beta} \right)^2. \quad (7.4)$$

From the input data given in Table II, we compute the parameter values shown in Table III, since the transformed  $X$  force-level equation is given by (3.9) with  $q = (v + 1)/(\mu + v + 2)$ ,  $p = 1 - q$ ,  $\mu = 1$ , and  $v = 2$ . Thus, the  $X$  force level may be computed with  $F_\alpha(\tau)$  and  $H_{1-\alpha}(\tau)$  with  $\alpha = q = 3/5$ . Force-annihilation prediction involves the limiting value of  $T_\alpha(\tau) = H_{1-\alpha}(\tau)/F_\alpha(\tau)$  as  $\tau \rightarrow +\infty$ . From Table 8B of the Appendix and Table III, we note the predicted agreement between  $\Gamma(1-\alpha)/\Gamma(\alpha)$  and the limiting value of  $T_\alpha(x)$  as  $x \rightarrow +\infty$  [recall (4.13)] for  $\alpha = q = 3/5$ . We now consider two cases: (I)  $R_0 = 2000$  meters, and (II)  $R_0 = 1250$  meters.

When  $R_0 = 2000$  meters (see Figure 3 of [4]), we have  $C = 0$  and  $\tau_0 = 0$ . The maximum time that the battle can last is  $t_{\max} = R_0/v = 14.91$  minutes, since at this time the attackers reach their final objective, i.e. the defender's position (again, see Figure 1). We now consider the qualitative behavior of the  $\mu = 1$ ,  $v = 2$  force-level trajectory shown in Figure 3 of [4]. Theorem 2 tells us that the  $X$  force can be annihilated in finite time if and only if

$$\frac{x_0}{y_0} < \frac{\Gamma(p)}{\Gamma(q)} \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + v + 2} \right)^{q-p}, \quad (7.3)$$

where  $q = 3/5$  and  $p = 1 - q$ . Using the numerical values in Table III, we compute from (7.3) that the  $X$  force can be annihilated in finite time if and only if



Table III. Parameter Values for Numerical Examples

$$k_a = 4.0233 \times 10^{-3} \text{ X casualties/minute}^u / \text{Y firer}$$

$$k_b = 2.6979 \times 10^{-3} \text{ Y casualties/minute}^v / \text{X firer}$$

$$p = 2/5, \quad q = 3/5$$

$$\Gamma(p)/\Gamma(q) = 1.48951$$

$$A = 0$$

$$\frac{x_0}{y_0} < 0.420 . \quad (7.4)$$

When the X force can be annihilated, its annihilation time is given by (5.8), which we rewrite here as

$$T_q(\tau(t_a^X)) = \frac{x_0}{y_0 \sqrt{\lambda_R}} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{p-q} , \quad (7.5)$$

where

$$\tau(t) = \left( \frac{2\lambda_I}{\mu + \nu + 2} \right) t^{(\mu+\nu+2)/2} . \quad (7.6)$$

Thus, for the numerical values given in Table III, the time of annihilation of the X force is given by

$$T_q(\tau(t_a^X)) = 3.544 \frac{x_0}{y_0} , \quad (7.7)$$

with  $q = 3/5$ . We will now illustrate further computations for  $x_0 = 10$  and  $y_0 = 30$ . From (7.4) we see that the X force can be annihilated in finite time (but we must verify that  $t_a^X \leq t_{\max}$ ). In this case (7.7) becomes

$$T_q(\tau(t_a^X)) = 1.18122 . \quad (7.8)$$

We must now determine  $\tau(t_a^X)$  such that  $\tau(t_a^X) = T_q^{-1}(1.18122)$  by using interpolation methods and Tables 8A and 8B. From Table 8A, we find

$$T_q(\tau) = 1.18172 \quad \text{for } \tau = 1.01$$

$$T_q(\tau) = 1.17630 \quad \text{for } \tau = 1.00$$

so that using linear interpolation, we obtain

$$\tau(t_a^X) = 1.009, \quad (7.9)$$

whence use of (7.6) yields

$$t_a^X = 14.24 \text{ minutes}, \quad (7.10)$$

which is less than  $t_{\max} = 14.91$  minutes so that the defending X force is indeed annihilated before the attacking Y force reaches its final objective.

Since  $r(t) = R_0 - vt$ , we find that force separation at the instant of annihilation of the X force is

$$r_a^X = 89.8 \text{ meters}. \quad (7.11)$$

Further results may be computed in a similar fashion and are given in Table IV.

When  $R_0 = 1250$  meters (see Figure 3 of [5]), we have  $C = 5.5923$  minutes,  $\tau_0 = 0.0975$ , and  $t_{\max} = R_0/v = 9.32$  minutes. In this case Theorem 2 tells us that the X force can be annihilated in finite time if and only if



Table IV. Annihilation of the X Force as a Function  
of the Initial Force Ratio for  $R_0 = 2000$  meters

$(x_0/y_0)$	$t_a^X$ (minutes)	$r_a^X$ (meters)
0.333	14.24	89.8
0.250	11.61	443.2
0.200	10.19	633.2

$$\frac{x_0}{y_0} < \sqrt{\lambda_R} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{q-p} \frac{\Gamma(p)}{\Gamma(q)} \frac{\left\{ F_q(\tau_0) - \frac{\Gamma(q)}{\Gamma(p)} H_p(\tau_0) \right\}}{\left\{ F_p(\tau_0) - \frac{\Gamma(p)}{\Gamma(q)} H_q(\tau_0) \right\}}, \quad (7.12)$$

with  $q = 3/5$  and  $p = 1-q$ . Using linear interpolation, we obtain from Tables 7A and 8A of the Appendix that for the numerical values of Table III

$$F_p(\tau_0) = 1.006, \quad H_q(\tau_0) = 0.044, \quad (7.13)$$

$$F_q(\tau_0) = 1.004, \quad H_p(\tau_0) = 0.223,$$

so that (7.12) says that the  $X$  force can be annihilated if and only if

$$\frac{x_0}{y_0} < 0.382. \quad (7.14)$$

When the  $X$  force can be annihilated, its annihilation time is given by (5.4), which we rewrite here as

$$T_q(\tau(t_a^X)) = \frac{\left\{ \frac{x_0}{y_0 \sqrt{\lambda_R}} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{p-q} F_p(\tau_0) + H_p(\tau_0) \right\}}{\left\{ F_q(\tau_0) + \frac{x_0}{y_0 \sqrt{\lambda_R}} \left( \frac{\lambda_I}{\mu + \nu + 2} \right)^{p-q} H_q(\tau_0) \right\}}, \quad (7.15)$$

whence for the data of Table III

$$T_a(\tau(t_a^X)) = \frac{3.565u_0 + 0.223}{0.156u_0 + 1.004}, \quad (7.16)$$

where  $u_0 = x_0/y_0$ . Let us also record here that (3.11) yields

$$t = \left( \frac{\{\mu + \nu + 2\}\tau}{2\lambda_I} \right)^{2/(\mu+\nu+2)} - C. \quad (7.17)$$

We will again illustrate further computations for  $x_0 = 10$  and  $y_0 = 30$ .

From (7.14) we see that the  $X$  force can be annihilated in finite time (but again we must investigate whether or not  $t_a^X \leq t_{\max}$ ). In this case (7.16) becomes

$$T_q(\tau(t_a^X)) = 1.33651, \quad (7.18)$$

whence Table 8A of the Appendix and linear interpolation yield

$$\tau(t_a^X) = 1.397, \quad (7.19)$$

so that by (7.17)

$$t_a^X = 10.63 \text{ minutes}. \quad (7.20)$$

Since  $t_{\max} = R_0/v = 9.32 \text{ minutes} < t_a^X$ , we see that the attacking  $Y$  force overruns the defender's position before annihilation of the  $X$  force occurs.

Thus, the battle ends with  $x_f = x(t_f) > 0$  and  $y_f > 0$  at  $t_f = t_{\max} = 9.32 \text{ minutes}$ . Corresponding to  $t_f = 9.32 \text{ minutes}$  is  $\tau_f = 1.1318$ , and then Table 8A of the Appendix yields



$$F_q(\tau_f = 1.1318) = 1.589, \quad H_p(1.1318) = 1.973, \quad (7.21)$$

whence via (2.4), (4.8), (4.9), and (7.13) we obtain

$$x_f = x(t_f) = x(r = 0) = 1.35. \quad (7.22)$$

Some further numerical results are given in Table V. Again, these parametric results should be contrasted with the single  $\mu = 1, \nu = 2$  force-level trajectory shown in Figure 3 of [5].

## 8. Final Remarks

In the previous section above, we have seen how the LCS functions allow one to conveniently obtain much valuable information about the model (2.1) with power attrition-rate coefficients (3.8) without having to explicitly compute the entire force-level trajectories. Previously we were limited to computing only force-level trajectories (see [4-5]). With the availability of these tabulations of LCS functions (see the Appendix of this report and [8]), we can now tell who is going to be annihilated and when this event will happen without having to compute the trajectories. Not only did we answer questions about the qualitative behavior of the model (e.g. force annihilation) for specific values of, for example, initial force levels but also for a range of values of the initial force ratio (i.e. parametric analysis of model behavior).

Table V. Annihilation of the X Force as a Function  
of the Initial Force Ratio for  $R_0 = 1250$  meters

$(x_0/y_0)$	$t_a^X$ (minutes)	$r_a^X$ (meters)
0.333	10.63	<sup>†</sup>
0.250	7.56	235.9
0.200	6.17	422.8

$$^{\dagger}t_{\max} = 9.32 \text{ minutes and } x_f = x(r=0) = 1.35.$$

The results of this report may be used for other parametric analyses, e.g. parametric dependence of battle outcome on attrition-rate coefficients. Thus, the contents of this report allow one to develop important insights into the dynamics of combat between two homogeneous forces with temporal variations in fire effectiveness. With the availability of tabulations of the LCS functions, one can now analyze such combat modelled by the power attrition-rate coefficients (3.8) with somewhat the same facility as he can for the constant-coefficient case and thus aid in parametric analyses. For further discussions of the significance of such results for military operations research, the reader is directed to [6] and [7].



## REFERENCES

1. M. ABRAMOWITZ and I. STEGUN (Editors), Handbook of Mathematical Functions, National Bureau of Standards Applied Mathematics Series, No. 55, Washington, D.C., 1964.
2. K. KNOPP, Infinite Sequences and Series, Dover, New York, 1956.
3. F. W. LANCHESTER, "Aircraft in Warfare: The Dawn of the Fourth Arm-  
No. V., The Principle of Concentration," Engineering 98, 422-423 (1914)  
(reprinted on pp. 2138-2148 of The World of Mathematics, J. Newman  
(Editor), Simon and Schuster, New York, 1956).
4. J. G. TAYLOR, "Solving Lanchester-Type Equations for 'Modern Warfare'  
with Variable Coefficients," Opns. Res. 22, 756-770 (1974).
5. J. G. TAYLOR and G. G. BROWN, "Canonical Methods in the Solution of  
Variable-Coefficient Lanchester-Type Equations of Modern Warfare,"  
Opns. Res. 24, 44-69 (1976).
6. J. G. TAYLOR and G. G. BROWN, "Further Canonical Methods in the  
Solution of Variable-Coefficient Lanchester-Type Equations of Modern  
Warfare," NPS55-77-27, Naval Postgraduate School, Monterey, California,  
June 1977.
7. J. G. TAYLOR and C. COMSTOCK, "Force-Annihilation Conditions for  
Variable-Coefficient Lanchester-Type Equations of Modern Warfare,"  
Naval Res. Log. Quart. 24, 349-371 (1977).
8. J. G. TAYLOR and G. G. BROWN, "A Short Table of Lanchester-Clifford-  
Schläfli Functions," NPS55-77-42, Naval Postgraduate School, Monterey,  
California, October 1977.

**APPENDIX: Tabulation of the LCS Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and  $T_{\alpha}(x)$  for**  
 $\alpha = 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 2/7, 3/7,$   
 $4/7, 5/7, 4/9, 5/9, 3/11, 5/11, 6/11, 8/11, 5/13, 8/13,$   
 $5/17, 12/17, 5/21, \text{ and } 16/21.$

[illegible]

TABLE 1A. Lanchester-Clifford-Schläfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$T_{\alpha}(x)$  for  $\alpha = 1/2$  and  $x$  from 0.00 to 1.50.





[illegible]

TABLE 2A. Lanchester-Clifford-Schläfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$$T_{\alpha}(x) \text{ for } \alpha = 1/3 \text{ and } x \text{ from } 0.00 \text{ to } 1.50.$$







$\alpha = 2/3$

$x$	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$	$x$	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$	$x$	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$
0.0000	1.0000	0.0000	0.0000	0.5000	0.9532	1.2411	1.3517	1.0000	0.0000	2.2370	1.1230
0.0005	1.0001	0.0000	0.0000	0.5005	1.0037	1.2403	1.3514	1.0005	0.0001	2.2369	1.1230
0.0010	1.0002	0.0000	0.0000	0.5010	1.0542	1.2395	1.3511	1.0010	0.0002	2.2368	1.1230
0.0015	1.0003	0.0000	0.0000	0.5015	1.1047	1.2387	1.3508	1.0015	0.0003	2.2367	1.1230
0.0020	1.0004	0.0000	0.0000	0.5020	1.1552	1.2379	1.3505	1.0020	0.0004	2.2366	1.1230
0.0025	1.0005	0.0000	0.0000	0.5025	1.2057	1.2371	1.3502	1.0025	0.0005	2.2365	1.1230
0.0030	1.0006	0.0000	0.0000	0.5030	1.2562	1.2363	1.3499	1.0030	0.0006	2.2364	1.1230
0.0035	1.0007	0.0000	0.0000	0.5035	1.3067	1.2355	1.3496	1.0035	0.0007	2.2363	1.1230
0.0040	1.0008	0.0000	0.0000	0.5040	1.3572	1.2347	1.3493	1.0040	0.0008	2.2362	1.1230
0.0045	1.0009	0.0000	0.0000	0.5045	1.4077	1.2339	1.3490	1.0045	0.0009	2.2361	1.1230
0.0050	1.0010	0.0000	0.0000	0.5050	1.4582	1.2331	1.3487	1.0050	0.0010	2.2360	1.1230
0.0055	1.0011	0.0000	0.0000	0.5055	1.5087	1.2323	1.3484	1.0055	0.0011	2.2359	1.1230
0.0060	1.0012	0.0000	0.0000	0.5060	1.5592	1.2315	1.3481	1.0060	0.0012	2.2358	1.1230
0.0065	1.0013	0.0000	0.0000	0.5065	1.6097	1.2307	1.3478	1.0065	0.0013	2.2357	1.1230
0.0070	1.0014	0.0000	0.0000	0.5070	1.6602	1.2299	1.3475	1.0070	0.0014	2.2356	1.1230
0.0075	1.0015	0.0000	0.0000	0.5075	1.7107	1.2291	1.3472	1.0075	0.0015	2.2355	1.1230
0.0080	1.0016	0.0000	0.0000	0.5080	1.7612	1.2283	1.3469	1.0080	0.0016	2.2354	1.1230
0.0085	1.0017	0.0000	0.0000	0.5085	1.8117	1.2275	1.3466	1.0085	0.0017	2.2353	1.1230
0.0090	1.0018	0.0000	0.0000	0.5090	1.8622	1.2267	1.3463	1.0090	0.0018	2.2352	1.1230
0.0095	1.0019	0.0000	0.0000	0.5095	1.9127	1.2259	1.3460	1.0095	0.0019	2.2351	1.1230
0.0100	1.0020	0.0000	0.0000	0.5100	1.9632	1.2251	1.3457	1.0100	0.0020	2.2350	1.1230
0.0105	1.0021	0.0000	0.0000	0.5105	2.0137	1.2243	1.3454	1.0105	0.0021	2.2349	1.1230
0.0110	1.0022	0.0000	0.0000	0.5110	2.0642	1.2235	1.3451	1.0110	0.0022	2.2348	1.1230
0.0115	1.0023	0.0000	0.0000	0.5115	2.1147	1.2227	1.3448	1.0115	0.0023	2.2347	1.1230
0.0120	1.0024	0.0000	0.0000	0.5120	2.1652	1.2219	1.3445	1.0120	0.0024	2.2346	1.1230
0.0125	1.0025	0.0000	0.0000	0.5125	2.2157	1.2211	1.3442	1.0125	0.0025	2.2345	1.1230
0.0130	1.0026	0.0000	0.0000	0.5130	2.2662	1.2203	1.3439	1.0130	0.0026	2.2344	1.1230
0.0135	1.0027	0.0000	0.0000	0.5135	2.3167	1.2195	1.3436	1.0135	0.0027	2.2343	1.1230
0.0140	1.0028	0.0000	0.0000	0.5140	2.3672	1.2187	1.3433	1.0140	0.0028	2.2342	1.1230
0.0145	1.0029	0.0000	0.0000	0.5145	2.4177	1.2179	1.3430	1.0145	0.0029	2.2341	1.1230
0.0150	1.0030	0.0000	0.0000	0.5150	2.4682	1.2171	1.3427	1.0150	0.0030	2.2340	1.1230
0.0155	1.0031	0.0000	0.0000	0.5155	2.5187	1.2163	1.3424	1.0155	0.0031	2.2339	1.1230
0.0160	1.0032	0.0000	0.0000	0.5160	2.5692	1.2155	1.3421	1.0160	0.0032	2.2338	1.1230
0.0165	1.0033	0.0000	0.0000	0.5165	2.6197	1.2147	1.3418	1.0165	0.0033	2.2337	1.1230
0.0170	1.0034	0.0000	0.0000	0.5170	2.6702	1.2139	1.3415	1.0170	0.0034	2.2336	1.1230
0.0175	1.0035	0.0000	0.0000	0.5175	2.7207	1.2131	1.3412	1.0175	0.0035	2.2335	1.1230
0.0180	1.0036	0.0000	0.0000	0.5180	2.7712	1.2123	1.3409	1.0180	0.0036	2.2334	1.1230
0.0185	1.0037	0.0000	0.0000	0.5185	2.8217	1.2115	1.3406	1.0185	0.0037	2.2333	1.1230
0.0190	1.0038	0.0000	0.0000	0.5190	2.8722	1.2107	1.3403	1.0190	0.0038	2.2332	1.1230
0.0195	1.0039	0.0000	0.0000	0.5195	2.9227	1.2099	1.3400	1.0195	0.0039	2.2331	1.1230
0.0200	1.0040	0.0000	0.0000	0.5200	2.9732	1.2091	1.3397	1.0200	0.0040	2.2330	1.1230
0.0205	1.0041	0.0000	0.0000	0.5205	3.0237	1.2083	1.3394	1.0205	0.0041	2.2329	1.1230
0.0210	1.0042	0.0000	0.0000	0.5210	3.0742	1.2075	1.3391	1.0210	0.0042	2.2328	1.1230
0.0215	1.0043	0.0000	0.0000	0.5215	3.1247	1.2067	1.3388	1.0215	0.0043	2.2327	1.1230
0.0220	1.0044	0.0000	0.0000	0.5220	3.1752	1.2059	1.3385	1.0220	0.0044	2.2326	1.1230
0.0225	1.0045	0.0000	0.0000	0.5225	3.2257	1.2051	1.3382	1.0225	0.0045	2.2325	1.1230
0.0230	1.0046	0.0000	0.0000	0.5230	3.2762	1.2043	1.3379	1.0230	0.0046	2.2324	1.1230
0.0235	1.0047	0.0000	0.0000	0.5235	3.3267	1.2035	1.3376	1.0235	0.0047	2.2323	1.1230
0.0240	1.0048	0.0000	0.0000	0.5240	3.3772	1.2027	1.3373	1.0240	0.0048	2.2322	1.1230
0.0245	1.0049	0.0000	0.0000	0.5245	3.4277	1.2019	1.3370	1.0245	0.0049	2.2321	1.1230
0.0250	1.0050	0.0000	0.0000	0.5250	3.4782	1.2011	1.3367	1.0250	0.0050	2.2320	1.1230
0.0255	1.0051	0.0000	0.0000	0.5255	3.5287	1.2003	1.3364	1.0255	0.0051	2.2319	1.1230
0.0260	1.0052	0.0000	0.0000	0.5260	3.5792	1.1995	1.3361	1.0260	0.0052	2.2318	1.1230
0.0265	1.0053	0.0000	0.0000	0.5265	3.6297	1.1987	1.3358	1.0265	0.0053	2.2317	1.1230
0.0270	1.0054	0.0000	0.0000	0.5270	3.6802	1.1979	1.3355	1.0270	0.0054	2.2316	1.1230
0.0275	1.0055	0.0000	0.0000	0.5275	3.7307	1.1971	1.3352	1.0275	0.0055	2.2315	1.1230
0.0280	1.0056	0.0000	0.0000	0.5280	3.7812	1.1963	1.3349	1.0280	0.0056	2.2314	1.1230
0.0285	1.0057	0.0000	0.0000	0.5285	3.8317	1.1955	1.3346	1.0285	0.0057	2.2313	1.1230
0.0290	1.0058	0.0000	0.0000	0.5290	3.8822	1.1947	1.3343	1.0290	0.0058	2.2312	1.1230
0.0295	1.0059	0.0000	0.0000	0.5295	3.9327	1.1939	1.3340	1.0295	0.0059	2.2311	1.1230
0.0300	1.0060	0.0000	0.0000	0.5300	3.9832	1.1931	1.3337	1.0300	0.0060	2.2310	1.1230
0.0305	1.0061	0.0000	0.0000	0.5305	4.0337	1.1923	1.3334	1.0305	0.0061	2.2309	1.1230
0.0310	1.0062	0.0000	0.0000	0.5310	4.0842	1.1915	1.3331	1.0310	0.0062	2.2308	1.1230
0.0315	1.0063	0.0000	0.0000	0.5315	4.1347	1.1907	1.3328	1.0315	0.0063	2.2307	1.1230
0.0320	1.0064	0.0000	0.0000	0.5320	4.1852	1.1899	1.3325	1.0320	0.0064	2.2306	1.1230
0.0325	1.0065	0.0000	0.0000	0.5325	4.2357	1.1891	1.3322	1.0325	0.0065	2.2305	1.1230
0.0330	1.0066	0.0000	0.0000	0.5330	4.2862	1.1883	1.3319	1.0330	0.0066	2.2304	1.1230
0.0335	1.0067	0.0000	0.0000	0.5335	4.3367	1.1875	1.3316	1.0335	0.0067	2.2303	1.1230
0.0340	1.0068	0.0000	0.0000	0.5340	4.3872	1.1867	1.3313	1.0340	0.0068	2.2302	1.1230
0.0345	1.0069	0.0000	0.0000	0.5345	4.4377	1.1859	1.3310	1.0345	0.0069	2.2301	1.1230
0.0350	1.0070	0.0000	0.0000	0.5350	4.4882	1.1851	1.3307	1.0350	0.0070	2.2300	1.1230
0.0355	1.0071	0.0000	0.0000	0.5355	4.5387	1.1843	1.3304	1.0355	0.0071	2.2299	1.1230
0.0360	1.0072	0.0000	0.0000	0.5360	4.5892	1.1835	1.3301	1.0360	0.0072	2.2298	1.1230
0.0365	1.0073	0.0000	0.0000	0.5365	4.6397	1.1827	1.3298	1.0365	0.0073	2.2297	1.1230
0.0370	1.0074	0.0000	0.0000	0.5370	4.6902	1.1819	1.3295	1.0370	0.0074	2.2296	1.1230
0.0375	1.0075	0.0000	0.0000	0.5375	4.7407	1.1811	1.3292	1.0375	0.0075	2.2295	1.1230
0.0380	1.0076	0.0000	0.0000	0.5380	4.7912	1.1803	1.3289	1.0380	0.0076	2.2294	1.1230
0.0385	1.0077	0.0000	0.0000	0.5385	4.8417	1.1795	1.3286	1.0385	0.0077	2.2293	1.1230
0.0390	1.0078	0.0000	0.0000	0.5390	4.8922	1.1787	1.3283	1.0390	0.0078	2.2292	1.1230
0.0395	1.0079	0.0000	0.0000	0.5395	4.9427	1.1779	1.3280	1.0395	0.0079	2.2291	1.1230
0.0400	1.0080	0.0000	0.0000	0.5400	4.9932	1.1771	1.3277	1.0400	0.0080	2.2290	1.1230
0.0405	1.0081	0.0000	0.0000	0.5405	5.0437	1.1763	1.3274	1.0405	0.0081	2.2289	1.1230
0.0410	1.0082	0.0000	0.0000	0.5410	5.0942	1.1755	1.3271	1.0410	0.0082	2.2288	1.1230
0.0415	1.0083	0.0000	0.0000	0.5415	5.1447	1.1747	1.3268	1.0415	0.0083	2.2287	1.1230
0.0420	1.0084	0.0000	0.0000	0.5420	5.1952	1.1739	1.3265	1.0420	0.0084	2.2286	1.1230
0.0425	1.0085	0.0000	0.0000	0.5425	5.2457	1.1731	1.3262	1.0425	0.0085	2.2285	1.1230
0.0430	1.0086	0.0000	0.0000	0.5430	5.2962	1.1723	1.3259	1.0430	0.0086	2.2284	1.1230
0.0435	1.0087	0.0000	0.0000	0.5435	5.3467	1.1715	1.3256	1.0435	0.0087	2.2283	1.1230
0.0440	1.0088	0.0000	0.0000	0.5440	5.3972	1.1707	1.				

$\alpha = 2/3$ 

$x$	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$	$x$	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$	$x$	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$
1.0000	0.0000	0.0000	1.0000	6.0	129.97145	5.78225	1.92118	6.0	129.97145	5.78225	1.92118
1.0001	0.0000	0.0000	1.0000	6.1	129.97145	5.78225	1.92118	6.1	129.97145	5.78225	1.92118
1.0002	0.0000	0.0000	1.0000	6.2	129.97145	5.78225	1.92118	6.2	129.97145	5.78225	1.92118
1.0003	0.0000	0.0000	1.0000	6.3	129.97145	5.78225	1.92118	6.3	129.97145	5.78225	1.92118
1.0004	0.0000	0.0000	1.0000	6.4	129.97145	5.78225	1.92118	6.4	129.97145	5.78225	1.92118
1.0005	0.0000	0.0000	1.0000	6.5	129.97145	5.78225	1.92118	6.5	129.97145	5.78225	1.92118
1.0006	0.0000	0.0000	1.0000	6.6	129.97145	5.78225	1.92118	6.6	129.97145	5.78225	1.92118
1.0007	0.0000	0.0000	1.0000	6.7	129.97145	5.78225	1.92118	6.7	129.97145	5.78225	1.92118
1.0008	0.0000	0.0000	1.0000	6.8	129.97145	5.78225	1.92118	6.8	129.97145	5.78225	1.92118
1.0009	0.0000	0.0000	1.0000	6.9	129.97145	5.78225	1.92118	6.9	129.97145	5.78225	1.92118
1.0010	0.0000	0.0000	1.0000	7.0	129.97145	5.78225	1.92118	7.0	129.97145	5.78225	1.92118
1.0011	0.0000	0.0000	1.0000	7.1	129.97145	5.78225	1.92118	7.1	129.97145	5.78225	1.92118
1.0012	0.0000	0.0000	1.0000	7.2	129.97145	5.78225	1.92118	7.2	129.97145	5.78225	1.92118
1.0013	0.0000	0.0000	1.0000	7.3	129.97145	5.78225	1.92118	7.3	129.97145	5.78225	1.92118
1.0014	0.0000	0.0000	1.0000	7.4	129.97145	5.78225	1.92118	7.4	129.97145	5.78225	1.92118
1.0015	0.0000	0.0000	1.0000	7.5	129.97145	5.78225	1.92118	7.5	129.97145	5.78225	1.92118
1.0016	0.0000	0.0000	1.0000	7.6	129.97145	5.78225	1.92118	7.6	129.97145	5.78225	1.92118
1.0017	0.0000	0.0000	1.0000	7.7	129.97145	5.78225	1.92118	7.7	129.97145	5.78225	1.92118
1.0018	0.0000	0.0000	1.0000	7.8	129.97145	5.78225	1.92118	7.8	129.97145	5.78225	1.92118
1.0019	0.0000	0.0000	1.0000	7.9	129.97145	5.78225	1.92118	7.9	129.97145	5.78225	1.92118
1.0020	0.0000	0.0000	1.0000	8.0	129.97145	5.78225	1.92118	8.0	129.97145	5.78225	1.92118
1.0021	0.0000	0.0000	1.0000	8.1	129.97145	5.78225	1.92118	8.1	129.97145	5.78225	1.92118
1.0022	0.0000	0.0000	1.0000	8.2	129.97145	5.78225	1.92118	8.2	129.97145	5.78225	1.92118
1.0023	0.0000	0.0000	1.0000	8.3	129.97145	5.78225	1.92118	8.3	129.97145	5.78225	1.92118
1.0024	0.0000	0.0000	1.0000	8.4	129.97145	5.78225	1.92118	8.4	129.97145	5.78225	1.92118
1.0025	0.0000	0.0000	1.0000	8.5	129.97145	5.78225	1.92118	8.5	129.97145	5.78225	1.92118
1.0026	0.0000	0.0000	1.0000	8.6	129.97145	5.78225	1.92118	8.6	129.97145	5.78225	1.92118
1.0027	0.0000	0.0000	1.0000	8.7	129.97145	5.78225	1.92118	8.7	129.97145	5.78225	1.92118
1.0028	0.0000	0.0000	1.0000	8.8	129.97145	5.78225	1.92118	8.8	129.97145	5.78225	1.92118
1.0029	0.0000	0.0000	1.0000	8.9	129.97145	5.78225	1.92118	8.9	129.97145	5.78225	1.92118
1.0030	0.0000	0.0000	1.0000	9.0	129.97145	5.78225	1.92118	9.0	129.97145	5.78225	1.92118
1.0031	0.0000	0.0000	1.0000	9.1	129.97145	5.78225	1.92118	9.1	129.97145	5.78225	1.92118
1.0032	0.0000	0.0000	1.0000	9.2	129.97145	5.78225	1.92118	9.2	129.97145	5.78225	1.92118
1.0033	0.0000	0.0000	1.0000	9.3	129.97145	5.78225	1.92118	9.3	129.97145	5.78225	1.92118
1.0034	0.0000	0.0000	1.0000	9.4	129.97145	5.78225	1.92118	9.4	129.97145	5.78225	1.92118
1.0035	0.0000	0.0000	1.0000	9.5	129.97145	5.78225	1.92118	9.5	129.97145	5.78225	1.92118
1.0036	0.0000	0.0000	1.0000	9.6	129.97145	5.78225	1.92118	9.6	129.97145	5.78225	1.92118
1.0037	0.0000	0.0000	1.0000	9.7	129.97145	5.78225	1.92118	9.7	129.97145	5.78225	1.92118
1.0038	0.0000	0.0000	1.0000	9.8	129.97145	5.78225	1.92118	9.8	129.97145	5.78225	1.92118
1.0039	0.0000	0.0000	1.0000	9.9	129.97145	5.78225	1.92118	9.9	129.97145	5.78225	1.92118
1.0040	0.0000	0.0000	1.0000	10.0	129.97145	5.78225	1.92118	10.0	129.97145	5.78225	1.92118

TABLE 3B. Lanchester-Clifford-Schlafli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and $T_{\alpha}(x)$  for  $\alpha = 2/3$  and  $x$  from 1.50 to 10.0.









$\alpha = 3/4$

$x$	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$	$x$	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$	$x$	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.3588	3.4234	2.5332
0.0005	1.0001	0.0000	0.0000	0.0005	0.0001	0.0000	0.0000	0.9995	1.3559	3.4230	2.5331
0.0010	1.0002	0.0000	0.0000	0.0010	0.0002	0.0000	0.0000	0.9990	1.3531	3.4225	2.5330
0.0015	1.0003	0.0000	0.0000	0.0015	0.0003	0.0000	0.0000	0.9985	1.3503	3.4220	2.5329
0.0020	1.0004	0.0000	0.0000	0.0020	0.0004	0.0000	0.0000	0.9980	1.3475	3.4215	2.5328
0.0025	1.0005	0.0000	0.0000	0.0025	0.0005	0.0000	0.0000	0.9975	1.3447	3.4210	2.5327
0.0030	1.0006	0.0000	0.0000	0.0030	0.0006	0.0000	0.0000	0.9970	1.3419	3.4205	2.5326
0.0035	1.0007	0.0000	0.0000	0.0035	0.0007	0.0000	0.0000	0.9965	1.3391	3.4200	2.5325
0.0040	1.0008	0.0000	0.0000	0.0040	0.0008	0.0000	0.0000	0.9960	1.3363	3.4195	2.5324
0.0045	1.0009	0.0000	0.0000	0.0045	0.0009	0.0000	0.0000	0.9955	1.3335	3.4190	2.5323
0.0050	1.0010	0.0000	0.0000	0.0050	0.0010	0.0000	0.0000	0.9950	1.3307	3.4185	2.5322
0.0055	1.0011	0.0000	0.0000	0.0055	0.0011	0.0000	0.0000	0.9945	1.3279	3.4180	2.5321
0.0060	1.0012	0.0000	0.0000	0.0060	0.0012	0.0000	0.0000	0.9940	1.3251	3.4175	2.5320
0.0065	1.0013	0.0000	0.0000	0.0065	0.0013	0.0000	0.0000	0.9935	1.3223	3.4170	2.5319
0.0070	1.0014	0.0000	0.0000	0.0070	0.0014	0.0000	0.0000	0.9930	1.3195	3.4165	2.5318
0.0075	1.0015	0.0000	0.0000	0.0075	0.0015	0.0000	0.0000	0.9925	1.3167	3.4160	2.5317
0.0080	1.0016	0.0000	0.0000	0.0080	0.0016	0.0000	0.0000	0.9920	1.3139	3.4155	2.5316
0.0085	1.0017	0.0000	0.0000	0.0085	0.0017	0.0000	0.0000	0.9915	1.3111	3.4150	2.5315
0.0090	1.0018	0.0000	0.0000	0.0090	0.0018	0.0000	0.0000	0.9910	1.3083	3.4145	2.5314
0.0095	1.0019	0.0000	0.0000	0.0095	0.0019	0.0000	0.0000	0.9905	1.3055	3.4140	2.5313
0.0100	1.0020	0.0000	0.0000	0.0100	0.0020	0.0000	0.0000	0.9900	1.3027	3.4135	2.5312
0.0105	1.0021	0.0000	0.0000	0.0105	0.0021	0.0000	0.0000	0.9895	1.2999	3.4130	2.5311
0.0110	1.0022	0.0000	0.0000	0.0110	0.0022	0.0000	0.0000	0.9890	1.2971	3.4125	2.5310
0.0115	1.0023	0.0000	0.0000	0.0115	0.0023	0.0000	0.0000	0.9885	1.2943	3.4120	2.5309
0.0120	1.0024	0.0000	0.0000	0.0120	0.0024	0.0000	0.0000	0.9880	1.2915	3.4115	2.5308
0.0125	1.0025	0.0000	0.0000	0.0125	0.0025	0.0000	0.0000	0.9875	1.2887	3.4110	2.5307
0.0130	1.0026	0.0000	0.0000	0.0130	0.0026	0.0000	0.0000	0.9870	1.2859	3.4105	2.5306
0.0135	1.0027	0.0000	0.0000	0.0135	0.0027	0.0000	0.0000	0.9865	1.2831	3.4100	2.5305
0.0140	1.0028	0.0000	0.0000	0.0140	0.0028	0.0000	0.0000	0.9860	1.2803	3.4095	2.5304
0.0145	1.0029	0.0000	0.0000	0.0145	0.0029	0.0000	0.0000	0.9855	1.2775	3.4090	2.5303
0.0150	1.0030	0.0000	0.0000	0.0150	0.0030	0.0000	0.0000	0.9850	1.2747	3.4085	2.5302
0.0155	1.0031	0.0000	0.0000	0.0155	0.0031	0.0000	0.0000	0.9845	1.2719	3.4080	2.5301
0.0160	1.0032	0.0000	0.0000	0.0160	0.0032	0.0000	0.0000	0.9840	1.2691	3.4075	2.5300
0.0165	1.0033	0.0000	0.0000	0.0165	0.0033	0.0000	0.0000	0.9835	1.2663	3.4070	2.5299
0.0170	1.0034	0.0000	0.0000	0.0170	0.0034	0.0000	0.0000	0.9830	1.2635	3.4065	2.5298
0.0175	1.0035	0.0000	0.0000	0.0175	0.0035	0.0000	0.0000	0.9825	1.2607	3.4060	2.5297
0.0180	1.0036	0.0000	0.0000	0.0180	0.0036	0.0000	0.0000	0.9820	1.2579	3.4055	2.5296
0.0185	1.0037	0.0000	0.0000	0.0185	0.0037	0.0000	0.0000	0.9815	1.2551	3.4050	2.5295
0.0190	1.0038	0.0000	0.0000	0.0190	0.0038	0.0000	0.0000	0.9810	1.2523	3.4045	2.5294
0.0195	1.0039	0.0000	0.0000	0.0195	0.0039	0.0000	0.0000	0.9805	1.2495	3.4040	2.5293
0.0200	1.0040	0.0000	0.0000	0.0200	0.0040	0.0000	0.0000	0.9800	1.2467	3.4035	2.5292
0.0205	1.0041	0.0000	0.0000	0.0205	0.0041	0.0000	0.0000	0.9795	1.2439	3.4030	2.5291
0.0210	1.0042	0.0000	0.0000	0.0210	0.0042	0.0000	0.0000	0.9790	1.2411	3.4025	2.5290
0.0215	1.0043	0.0000	0.0000	0.0215	0.0043	0.0000	0.0000	0.9785	1.2383	3.4020	2.5289
0.0220	1.0044	0.0000	0.0000	0.0220	0.0044	0.0000	0.0000	0.9780	1.2355	3.4015	2.5288
0.0225	1.0045	0.0000	0.0000	0.0225	0.0045	0.0000	0.0000	0.9775	1.2327	3.4010	2.5287
0.0230	1.0046	0.0000	0.0000	0.0230	0.0046	0.0000	0.0000	0.9770	1.2299	3.4005	2.5286
0.0235	1.0047	0.0000	0.0000	0.0235	0.0047	0.0000	0.0000	0.9765	1.2271	3.4000	2.5285
0.0240	1.0048	0.0000	0.0000	0.0240	0.0048	0.0000	0.0000	0.9760	1.2243	3.3995	2.5284
0.0245	1.0049	0.0000	0.0000	0.0245	0.0049	0.0000	0.0000	0.9755	1.2215	3.3990	2.5283
0.0250	1.0050	0.0000	0.0000	0.0250	0.0050	0.0000	0.0000	0.9750	1.2187	3.3985	2.5282
0.0255	1.0051	0.0000	0.0000	0.0255	0.0051	0.0000	0.0000	0.9745	1.2159	3.3980	2.5281
0.0260	1.0052	0.0000	0.0000	0.0260	0.0052	0.0000	0.0000	0.9740	1.2131	3.3975	2.5280
0.0265	1.0053	0.0000	0.0000	0.0265	0.0053	0.0000	0.0000	0.9735	1.2103	3.3970	2.5279
0.0270	1.0054	0.0000	0.0000	0.0270	0.0054	0.0000	0.0000	0.9730	1.2075	3.3965	2.5278
0.0275	1.0055	0.0000	0.0000	0.0275	0.0055	0.0000	0.0000	0.9725	1.2047	3.3960	2.5277
0.0280	1.0056	0.0000	0.0000	0.0280	0.0056	0.0000	0.0000	0.9720	1.2019	3.3955	2.5276
0.0285	1.0057	0.0000	0.0000	0.0285	0.0057	0.0000	0.0000	0.9715	1.1991	3.3950	2.5275
0.0290	1.0058	0.0000	0.0000	0.0290	0.0058	0.0000	0.0000	0.9710	1.1963	3.3945	2.5274
0.0295	1.0059	0.0000	0.0000	0.0295	0.0059	0.0000	0.0000	0.9705	1.1935	3.3940	2.5273
0.0300	1.0060	0.0000	0.0000	0.0300	0.0060	0.0000	0.0000	0.9700	1.1907	3.3935	2.5272
0.0305	1.0061	0.0000	0.0000	0.0305	0.0061	0.0000	0.0000	0.9695	1.1879	3.3930	2.5271
0.0310	1.0062	0.0000	0.0000	0.0310	0.0062	0.0000	0.0000	0.9690	1.1851	3.3925	2.5270
0.0315	1.0063	0.0000	0.0000	0.0315	0.0063	0.0000	0.0000	0.9685	1.1823	3.3920	2.5269
0.0320	1.0064	0.0000	0.0000	0.0320	0.0064	0.0000	0.0000	0.9680	1.1795	3.3915	2.5268
0.0325	1.0065	0.0000	0.0000	0.0325	0.0065	0.0000	0.0000	0.9675	1.1767	3.3910	2.5267
0.0330	1.0066	0.0000	0.0000	0.0330	0.0066	0.0000	0.0000	0.9670	1.1739	3.3905	2.5266
0.0335	1.0067	0.0000	0.0000	0.0335	0.0067	0.0000	0.0000	0.9665	1.1711	3.3900	2.5265
0.0340	1.0068	0.0000	0.0000	0.0340	0.0068	0.0000	0.0000	0.9660	1.1683	3.3895	2.5264
0.0345	1.0069	0.0000	0.0000	0.0345	0.0069	0.0000	0.0000	0.9655	1.1655	3.3890	2.5263
0.0350	1.0070	0.0000	0.0000	0.0350	0.0070	0.0000	0.0000	0.9650	1.1627	3.3885	2.5262
0.0355	1.0071	0.0000	0.0000	0.0355	0.0071	0.0000	0.0000	0.9645	1.1599	3.3880	2.5261
0.0360	1.0072	0.0000	0.0000	0.0360	0.0072	0.0000	0.0000	0.9640	1.1571	3.3875	2.5260
0.0365	1.0073	0.0000	0.0000	0.0365	0.0073	0.0000	0.0000	0.9635	1.1543	3.3870	2.5259
0.0370	1.0074	0.0000	0.0000	0.0370	0.0074	0.0000	0.0000	0.9630	1.1515	3.3865	2.5258
0.0375	1.0075	0.0000	0.0000	0.0375	0.0075	0.0000	0.0000	0.9625	1.1487	3.3860	2.5257
0.0380	1.0076	0.0000	0.0000	0.0380	0.0076	0.0000	0.0000	0.9620	1.1459	3.3855	2.5256
0.0385	1.0077	0.0000	0.0000	0.0385	0.0077	0.0000	0.0000	0.9615	1.1431	3.3850	2.5255
0.0390	1.0078	0.0000	0.0000	0.0390	0.0078	0.0000	0.0000	0.9610	1.1403	3.3845	2.5254
0.0395	1.0079	0.0000	0.0000	0.0395	0.0079	0.0000	0.0000	0.9605	1.1375	3.3840	2.5253
0.0400	1.0080	0.0000	0.0000	0.0400	0.0080	0.0000	0.0000	0.9600	1.1347	3.3835	2.5252
0.0405	1.0081	0.0000	0.0000	0.0405	0.0081	0.0000	0.0000	0.9595	1.1319	3.3830	2.5251
0.0410	1.0082	0.0000	0.0000	0.0410	0.0082	0.0000	0.0000	0.9590	1.1291	3.3825	2.5250
0.0415	1.0083	0.0000	0.0000	0.0415	0.0083	0.0000	0.0000	0.9585	1.1263	3.3820	2.5249
0.0420	1.0084	0.0000	0.0000	0.0420	0.0084	0.0000	0.0000	0.9580	1.1235	3.3815	2.5248
0.0425	1.0085	0.0000	0.0000	0.0425	0.0085	0.0000	0.0000	0.9575	1.1207	3.3810	2.5247
0.0430	1.0086	0.0000	0.0000	0.0430	0.0086	0.0000	0.0000	0.9570	1.1179	3.3805	2.5246
0.0435	1.0087	0.0000	0.0000	0.0435	0.0087	0.0000	0.0000	0.9565	1.1151	3.3800	2.5245
0.0440	1.0088	0.0000	0.0000	0.0440	0.0088	0.0000	0.				









$\alpha = 1/5$

$x$	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$	$x$	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$	$x$	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$
0.00000	0.00000	1.00000	0.00000	2.00000	8.42066	2.03992	0.24688	4.00000	700.89071	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	9.42023	2.35895	0.24688	4.00000	778.66244	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	10.41979	2.68002	0.24688	4.00000	866.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	12.41820	3.00972	0.24688	4.00000	968.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	15.41639	3.43403	0.24688	4.00000	1086.88661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	17.41470	3.85403	0.24688	4.00000	1218.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	19.41302	4.26903	0.24688	4.00000	1364.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	21.41134	4.67903	0.24688	4.00000	1524.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	23.40966	5.08403	0.24688	4.00000	1698.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	25.40798	5.48403	0.24688	4.00000	1888.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	27.40630	5.87903	0.24688	4.00000	2093.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	29.40462	6.26903	0.24688	4.00000	2313.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	31.40294	6.65403	0.24688	4.00000	2547.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	33.40126	7.03403	0.24688	4.00000	2797.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	35.39958	7.40903	0.24688	4.00000	3062.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	37.39790	7.77903	0.24688	4.00000	3342.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	39.39622	8.14403	0.24688	4.00000	3636.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	41.39454	8.50403	0.24688	4.00000	3946.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	43.39286	8.85903	0.24688	4.00000	4271.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	45.39118	9.20903	0.24688	4.00000	4611.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	47.38950	9.55403	0.24688	4.00000	4966.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	49.38782	9.89403	0.24688	4.00000	5338.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	51.38614	10.22903	0.24688	4.00000	5726.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	53.38446	10.55903	0.24688	4.00000	6130.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	55.38278	10.88403	0.24688	4.00000	6550.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	57.38110	11.20403	0.24688	4.00000	6988.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	59.37942	11.51903	0.24688	4.00000	7443.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	61.37774	11.82903	0.24688	4.00000	7915.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	63.37606	12.13403	0.24688	4.00000	8404.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	65.37438	12.43403	0.24688	4.00000	8912.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	67.37270	12.72903	0.24688	4.00000	9438.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	69.37102	13.01903	0.24688	4.00000	9982.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	71.36934	13.30403	0.24688	4.00000	10544.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	73.36766	13.58403	0.24688	4.00000	11126.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	75.36598	13.85903	0.24688	4.00000	11727.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	77.36430	14.12903	0.24688	4.00000	12347.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	79.36262	14.39403	0.24688	4.00000	12985.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	81.36094	14.65403	0.24688	4.00000	13643.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	83.35926	14.90903	0.24688	4.00000	14320.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	85.35758	15.15903	0.24688	4.00000	15016.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	87.35590	15.40403	0.24688	4.00000	15731.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	89.35422	15.64403	0.24688	4.00000	16467.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	91.35254	15.87903	0.24688	4.00000	17223.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	93.35086	16.10903	0.24688	4.00000	18000.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	95.34918	16.33403	0.24688	4.00000	18797.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	97.34750	16.55403	0.24688	4.00000	19616.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	99.34582	16.76903	0.24688	4.00000	20456.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	101.34414	16.97903	0.24688	4.00000	21327.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	103.34246	17.18403	0.24688	4.00000	22229.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	105.34078	17.38403	0.24688	4.00000	23164.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	107.33910	17.57903	0.24688	4.00000	24132.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	109.33742	17.76903	0.24688	4.00000	25133.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	111.33574	17.95403	0.24688	4.00000	26167.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	113.33406	18.13403	0.24688	4.00000	27236.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	115.33238	18.30903	0.24688	4.00000	28339.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	117.33070	18.47903	0.24688	4.00000	29477.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	119.32902	18.64403	0.24688	4.00000	30650.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	121.32734	18.80403	0.24688	4.00000	31860.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	123.32566	18.95903	0.24688	4.00000	33106.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	125.32398	19.10903	0.24688	4.00000	34388.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	127.32230	19.25403	0.24688	4.00000	35706.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	129.32062	19.39403	0.24688	4.00000	37062.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	131.31894	19.52903	0.24688	4.00000	38455.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	133.31726	19.65903	0.24688	4.00000	39886.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	135.31558	19.78403	0.24688	4.00000	41355.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	137.31390	19.90403	0.24688	4.00000	42864.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	139.31222	20.01903	0.24688	4.00000	44412.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	141.31054	20.12903	0.24688	4.00000	45999.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	143.30886	20.23403	0.24688	4.00000	47625.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	145.30718	20.33403	0.24688	4.00000	49292.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	147.30550	20.42903	0.24688	4.00000	50999.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	149.30382	20.51903	0.24688	4.00000	52746.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	151.30214	20.60403	0.24688	4.00000	54533.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	153.30046	20.68403	0.24688	4.00000	56362.66704	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	155.29878	20.75903	0.24688	4.00000	58233.44062	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	157.29710	20.82903	0.24688	4.00000	60146.21427	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	159.29542	20.89403	0.24688	4.00000	62101.98661	17.74427	0.25360
0.00000	0.00000	1.00000	0.00000	2.00000	161.29374	20.95403	0.24688	4.00000	64100.66704	17.74427	0.25360



$\alpha = 2/5$

$x$	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$	$x$	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$	$x$	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$
0.0000	1.0000	0.0000	0.0000	0.5000	1.5977	0.2826	0.3807	1.0000	0.0000	0.0000	0.50173
0.0005	1.0005	0.0005	0.0005	0.5005	1.5971	0.2826	0.3807	1.0005	0.0005	0.0005	0.50183
0.0010	1.0010	0.0010	0.0010	0.5010	1.5964	0.2826	0.3807	1.0010	0.0010	0.0010	0.50193
0.0015	1.0015	0.0015	0.0015	0.5015	1.5957	0.2826	0.3807	1.0015	0.0015	0.0015	0.50203
0.0020	1.0020	0.0020	0.0020	0.5020	1.5950	0.2826	0.3807	1.0020	0.0020	0.0020	0.50213
0.0025	1.0025	0.0025	0.0025	0.5025	1.5943	0.2826	0.3807	1.0025	0.0025	0.0025	0.50223
0.0030	1.0030	0.0030	0.0030	0.5030	1.5936	0.2826	0.3807	1.0030	0.0030	0.0030	0.50233
0.0035	1.0035	0.0035	0.0035	0.5035	1.5929	0.2826	0.3807	1.0035	0.0035	0.0035	0.50243
0.0040	1.0040	0.0040	0.0040	0.5040	1.5922	0.2826	0.3807	1.0040	0.0040	0.0040	0.50253
0.0045	1.0045	0.0045	0.0045	0.5045	1.5915	0.2826	0.3807	1.0045	0.0045	0.0045	0.50263
0.0050	1.0050	0.0050	0.0050	0.5050	1.5908	0.2826	0.3807	1.0050	0.0050	0.0050	0.50273
0.0055	1.0055	0.0055	0.0055	0.5055	1.5901	0.2826	0.3807	1.0055	0.0055	0.0055	0.50283
0.0060	1.0060	0.0060	0.0060	0.5060	1.5894	0.2826	0.3807	1.0060	0.0060	0.0060	0.50293
0.0065	1.0065	0.0065	0.0065	0.5065	1.5887	0.2826	0.3807	1.0065	0.0065	0.0065	0.50303
0.0070	1.0070	0.0070	0.0070	0.5070	1.5880	0.2826	0.3807	1.0070	0.0070	0.0070	0.50313
0.0075	1.0075	0.0075	0.0075	0.5075	1.5873	0.2826	0.3807	1.0075	0.0075	0.0075	0.50323
0.0080	1.0080	0.0080	0.0080	0.5080	1.5866	0.2826	0.3807	1.0080	0.0080	0.0080	0.50333
0.0085	1.0085	0.0085	0.0085	0.5085	1.5859	0.2826	0.3807	1.0085	0.0085	0.0085	0.50343
0.0090	1.0090	0.0090	0.0090	0.5090	1.5852	0.2826	0.3807	1.0090	0.0090	0.0090	0.50353
0.0095	1.0095	0.0095	0.0095	0.5095	1.5845	0.2826	0.3807	1.0095	0.0095	0.0095	0.50363
0.0100	1.0100	0.0100	0.0100	0.5100	1.5838	0.2826	0.3807	1.0100	0.0100	0.0100	0.50373
0.0105	1.0105	0.0105	0.0105	0.5105	1.5831	0.2826	0.3807	1.0105	0.0105	0.0105	0.50383
0.0110	1.0110	0.0110	0.0110	0.5110	1.5824	0.2826	0.3807	1.0110	0.0110	0.0110	0.50393
0.0115	1.0115	0.0115	0.0115	0.5115	1.5817	0.2826	0.3807	1.0115	0.0115	0.0115	0.50403
0.0120	1.0120	0.0120	0.0120	0.5120	1.5810	0.2826	0.3807	1.0120	0.0120	0.0120	0.50413
0.0125	1.0125	0.0125	0.0125	0.5125	1.5803	0.2826	0.3807	1.0125	0.0125	0.0125	0.50423
0.0130	1.0130	0.0130	0.0130	0.5130	1.5796	0.2826	0.3807	1.0130	0.0130	0.0130	0.50433
0.0135	1.0135	0.0135	0.0135	0.5135	1.5789	0.2826	0.3807	1.0135	0.0135	0.0135	0.50443
0.0140	1.0140	0.0140	0.0140	0.5140	1.5782	0.2826	0.3807	1.0140	0.0140	0.0140	0.50453
0.0145	1.0145	0.0145	0.0145	0.5145	1.5775	0.2826	0.3807	1.0145	0.0145	0.0145	0.50463
0.0150	1.0150	0.0150	0.0150	0.5150	1.5768	0.2826	0.3807	1.0150	0.0150	0.0150	0.50473
0.0155	1.0155	0.0155	0.0155	0.5155	1.5761	0.2826	0.3807	1.0155	0.0155	0.0155	0.50483
0.0160	1.0160	0.0160	0.0160	0.5160	1.5754	0.2826	0.3807	1.0160	0.0160	0.0160	0.50493
0.0165	1.0165	0.0165	0.0165	0.5165	1.5747	0.2826	0.3807	1.0165	0.0165	0.0165	0.50503
0.0170	1.0170	0.0170	0.0170	0.5170	1.5740	0.2826	0.3807	1.0170	0.0170	0.0170	0.50513
0.0175	1.0175	0.0175	0.0175	0.5175	1.5733	0.2826	0.3807	1.0175	0.0175	0.0175	0.50523
0.0180	1.0180	0.0180	0.0180	0.5180	1.5726	0.2826	0.3807	1.0180	0.0180	0.0180	0.50533
0.0185	1.0185	0.0185	0.0185	0.5185	1.5719	0.2826	0.3807	1.0185	0.0185	0.0185	0.50543
0.0190	1.0190	0.0190	0.0190	0.5190	1.5712	0.2826	0.3807	1.0190	0.0190	0.0190	0.50553
0.0195	1.0195	0.0195	0.0195	0.5195	1.5705	0.2826	0.3807	1.0195	0.0195	0.0195	0.50563
0.0200	1.0200	0.0200	0.0200	0.5200	1.5698	0.2826	0.3807	1.0200	0.0200	0.0200	0.50573
0.0205	1.0205	0.0205	0.0205	0.5205	1.5691	0.2826	0.3807	1.0205	0.0205	0.0205	0.50583
0.0210	1.0210	0.0210	0.0210	0.5210	1.5684	0.2826	0.3807	1.0210	0.0210	0.0210	0.50593
0.0215	1.0215	0.0215	0.0215	0.5215	1.5677	0.2826	0.3807	1.0215	0.0215	0.0215	0.50603
0.0220	1.0220	0.0220	0.0220	0.5220	1.5670	0.2826	0.3807	1.0220	0.0220	0.0220	0.50613
0.0225	1.0225	0.0225	0.0225	0.5225	1.5663	0.2826	0.3807	1.0225	0.0225	0.0225	0.50623
0.0230	1.0230	0.0230	0.0230	0.5230	1.5656	0.2826	0.3807	1.0230	0.0230	0.0230	0.50633
0.0235	1.0235	0.0235	0.0235	0.5235	1.5649	0.2826	0.3807	1.0235	0.0235	0.0235	0.50643
0.0240	1.0240	0.0240	0.0240	0.5240	1.5642	0.2826	0.3807	1.0240	0.0240	0.0240	0.50653
0.0245	1.0245	0.0245	0.0245	0.5245	1.5635	0.2826	0.3807	1.0245	0.0245	0.0245	0.50663
0.0250	1.0250	0.0250	0.0250	0.5250	1.5628	0.2826	0.3807	1.0250	0.0250	0.0250	0.50673
0.0255	1.0255	0.0255	0.0255	0.5255	1.5621	0.2826	0.3807	1.0255	0.0255	0.0255	0.50683
0.0260	1.0260	0.0260	0.0260	0.5260	1.5614	0.2826	0.3807	1.0260	0.0260	0.0260	0.50693
0.0265	1.0265	0.0265	0.0265	0.5265	1.5607	0.2826	0.3807	1.0265	0.0265	0.0265	0.50703
0.0270	1.0270	0.0270	0.0270	0.5270	1.5600	0.2826	0.3807	1.0270	0.0270	0.0270	0.50713
0.0275	1.0275	0.0275	0.0275	0.5275	1.5593	0.2826	0.3807	1.0275	0.0275	0.0275	0.50723
0.0280	1.0280	0.0280	0.0280	0.5280	1.5586	0.2826	0.3807	1.0280	0.0280	0.0280	0.50733
0.0285	1.0285	0.0285	0.0285	0.5285	1.5579	0.2826	0.3807	1.0285	0.0285	0.0285	0.50743
0.0290	1.0290	0.0290	0.0290	0.5290	1.5572	0.2826	0.3807	1.0290	0.0290	0.0290	0.50753
0.0295	1.0295	0.0295	0.0295	0.5295	1.5565	0.2826	0.3807	1.0295	0.0295	0.0295	0.50763
0.0300	1.0300	0.0300	0.0300	0.5300	1.5558	0.2826	0.3807	1.0300	0.0300	0.0300	0.50773
0.0305	1.0305	0.0305	0.0305	0.5305	1.5551	0.2826	0.3807	1.0305	0.0305	0.0305	0.50783
0.0310	1.0310	0.0310	0.0310	0.5310	1.5544	0.2826	0.3807	1.0310	0.0310	0.0310	0.50793
0.0315	1.0315	0.0315	0.0315	0.5315	1.5537	0.2826	0.3807	1.0315	0.0315	0.0315	0.50803
0.0320	1.0320	0.0320	0.0320	0.5320	1.5530	0.2826	0.3807	1.0320	0.0320	0.0320	0.50813
0.0325	1.0325	0.0325	0.0325	0.5325	1.5523	0.2826	0.3807	1.0325	0.0325	0.0325	0.50823
0.0330	1.0330	0.0330	0.0330	0.5330	1.5516	0.2826	0.3807	1.0330	0.0330	0.0330	0.50833
0.0335	1.0335	0.0335	0.0335	0.5335	1.5509	0.2826	0.3807	1.0335	0.0335	0.0335	0.50843
0.0340	1.0340	0.0340	0.0340	0.5340	1.5502	0.2826	0.3807	1.0340	0.0340	0.0340	0.50853
0.0345	1.0345	0.0345	0.0345	0.5345	1.5495	0.2826	0.3807	1.0345	0.0345	0.0345	0.50863
0.0350	1.0350	0.0350	0.0350	0.5350	1.5488	0.2826	0.3807	1.0350	0.0350	0.0350	0.50873
0.0355	1.0355	0.0355	0.0355	0.5355	1.5481	0.2826	0.3807	1.0355	0.0355	0.0355	0.50883
0.0360	1.0360	0.0360	0.0360	0.5360	1.5474	0.2826	0.3807	1.0360	0.0360	0.0360	0.50893
0.0365	1.0365	0.0365	0.0365	0.5365	1.5467	0.2826	0.3807	1.0365	0.0365	0.0365	0.50903
0.0370	1.0370	0.0370	0.0370	0.5370	1.5460	0.2826	0.3807	1.0370	0.0370	0.0370	0.50913
0.0375	1.0375	0.0375	0.0375	0.5375	1.5453	0.2826	0.3807	1.0375	0.0375	0.0375	0.50923
0.0380	1.0380	0.0380	0.0380	0.5380	1.5446	0.2826	0.3807	1.0380	0.0380	0.0380	0.50933
0.0385	1.0385	0.0385	0.0385	0.5385	1.5439	0.2826	0.3807	1.0385	0.0385	0.0385	0.50943
0.0390	1.0390	0.0390	0.0390	0.5390	1.5432	0.2826	0.3807	1.0390	0.0390	0.0390	0.50953
0.0395	1.0395	0.0395	0.0395	0.5395	1.5425	0.2826	0.3807	1.0395	0.0395	0.0395	0.50963
0.0400	1.0400	0.0400	0.0400	0.5400	1.5418	0.2826	0.3807	1.0400	0.0400	0.0400	0.50973
0.0405	1.0405	0.0405	0.0405	0.5405	1.5411	0.2826	0.3807	1.0405	0.0405	0.0405	0.50983
0.0410	1.0410	0.0410	0.0410	0.5410	1.5404	0.2826	0.3807	1.0410	0.0410	0.0410	0.50993
0.0415	1.0415	0.0415	0.0415	0.5415	1.5397	0.2826	0.3807	1.0415	0.0415	0.0415	0.51003
0.0420	1.0420	0.0420	0.0420	0.5420	1.5390	0.2826	0.3807	1.0420	0.0420	0.0420	0.51013
0.0425	1.0425	0.0425	0.0425	0.5425	1.5383	0.2826	0.3807	1.0425	0.0425	0.0425	0.51023
0.0430	1.0430	0.0430	0.0430	0.5430	1.5376	0.2826	0.3807	1.0430	0.0430	0.0430	0.51033
0.0435	1.0435	0.0435	0.0435	0.5435	1.5369	0.2826	0.3807	1.0435	0.0435	0.0435	0.51043</



$\alpha = 2/5$ [illegible]

TABLE 7B. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and  $T_{\alpha}(x)$  for  $\alpha = 2/5$  and  $x$  from 1.50 to 10.0.









[illegible]

TABLE 9A. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and  $T_{\alpha}(x)$  for  $\alpha = 4/5$  and  $x$  from 0.00 to 1.50.

[illegible]



$\Gamma_{\alpha}(x)$  for  $\alpha = 2/7$  and  $x$  from 0.00 to 1.50.[illegible]





[illegible]

TABLE 11A. Lanchester-Clifford-Schläfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$$T_\alpha(x) \text{ for } \alpha = 3/7 \text{ and } x \text{ from } 0.00 \text{ to } 1.50.$$







$\alpha = 4/7$ 

$F_{4/7}(x)$	$F_{3/7}(x)$	$T_{4/7}(x)$	$x$	$F_{4/7}(x)$	$H_{3/7}(x)$	$T_{4/7}(x)$	$x$	$F_{4/7}(x)$	$H_{3/7}(x)$	$T_{4/7}(x)$
1.0000	0.0000	0.0000	0.0000	1.1517	0.7463	0.0000	1.0000	1.4745	2.5444	1.0521
1.0000	0.0000	0.0000	0.0000	1.2086	0.7769	0.0000	1.0000	1.4873	2.5544	1.0521
1.0000	0.0000	0.0000	0.0000	1.2366	0.7899	0.0000	1.0000	1.4914	2.5610	1.0521
1.0000	0.0000	0.0000	0.0000	1.2699	0.7999	0.0000	1.0000	1.5049	2.5800	1.0521
1.0000	0.0000	0.0000	0.0000	1.3066	0.8066	0.0000	1.0000	1.5157	2.5969	1.0521
1.0000	0.0000	0.0000	0.0000	1.3466	0.8111	0.0000	1.0000	1.5214	2.6099	1.0521
1.0000	0.0000	0.0000	0.0000	1.3899	0.8144	0.0000	1.0000	1.5249	2.6199	1.0521
1.0000	0.0000	0.0000	0.0000	1.4366	0.8166	0.0000	1.0000	1.5266	2.6266	1.0521
1.0000	0.0000	0.0000	0.0000	1.4866	0.8177	0.0000	1.0000	1.5277	2.6300	1.0521
1.0000	0.0000	0.0000	0.0000	1.5399	0.8177	0.0000	1.0000	1.5277	2.6300	1.0521
1.0000	0.0000	0.0000	0.0000	1.5966	0.8166	0.0000	1.0000	1.5266	2.6266	1.0521
1.0000	0.0000	0.0000	0.0000	1.6566	0.8144	0.0000	1.0000	1.5249	2.6199	1.0521
1.0000	0.0000	0.0000	0.0000	1.7199	0.8111	0.0000	1.0000	1.5214	2.6099	1.0521
1.0000	0.0000	0.0000	0.0000	1.7866	0.8066	0.0000	1.0000	1.5157	2.5969	1.0521
1.0000	0.0000	0.0000	0.0000	1.8566	0.8011	0.0000	1.0000	1.5089	2.5800	1.0521
1.0000	0.0000	0.0000	0.0000	1.9299	0.7944	0.0000	1.0000	1.5000	2.5610	1.0521
1.0000	0.0000	0.0000	0.0000	1.9966	0.7866	0.0000	1.0000	1.4899	2.5366	1.0521
1.0000	0.0000	0.0000	0.0000	2.0666	0.7777	0.0000	1.0000	1.4789	2.5099	1.0521
1.0000	0.0000	0.0000	0.0000	2.1399	0.7677	0.0000	1.0000	1.4666	2.4800	1.0521
1.0000	0.0000	0.0000	0.0000	2.2166	0.7566	0.0000	1.0000	1.4533	2.4466	1.0521
1.0000	0.0000	0.0000	0.0000	2.2966	0.7444	0.0000	1.0000	1.4399	2.4100	1.0521
1.0000	0.0000	0.0000	0.0000	2.3799	0.7311	0.0000	1.0000	1.4257	2.3711	1.0521
1.0000	0.0000	0.0000	0.0000	2.4666	0.7166	0.0000	1.0000	1.4111	2.3299	1.0521
1.0000	0.0000	0.0000	0.0000	2.5566	0.7011	0.0000	1.0000	1.3966	2.2866	1.0521
1.0000	0.0000	0.0000	0.0000	2.6499	0.6844	0.0000	1.0000	1.3822	2.2411	1.0521
1.0000	0.0000	0.0000	0.0000	2.7466	0.6666	0.0000	1.0000	1.3689	2.1933	1.0521
1.0000	0.0000	0.0000	0.0000	2.8466	0.6477	0.0000	1.0000	1.3566	2.1433	1.0521
1.0000	0.0000	0.0000	0.0000	2.9499	0.6277	0.0000	1.0000	1.3457	2.0911	1.0521
1.0000	0.0000	0.0000	0.0000	3.0566	0.6066	0.0000	1.0000	1.3366	2.0366	1.0521
1.0000	0.0000	0.0000	0.0000	3.1666	0.5844	0.0000	1.0000	1.3299	1.9800	1.0521
1.0000	0.0000	0.0000	0.0000	3.2799	0.5611	0.0000	1.0000	1.3257	1.9211	1.0521
1.0000	0.0000	0.0000	0.0000	3.3966	0.5366	0.0000	1.0000	1.3133	1.8566	1.0521
1.0000	0.0000	0.0000	0.0000	3.5166	0.5111	0.0000	1.0000	1.3022	1.7866	1.0521
1.0000	0.0000	0.0000	0.0000	3.6399	0.4844	0.0000	1.0000	1.2922	1.7111	1.0521
1.0000	0.0000	0.0000	0.0000	3.7666	0.4566	0.0000	1.0000	1.2833	1.6311	1.0521
1.0000	0.0000	0.0000	0.0000	3.8966	0.4277	0.0000	1.0000	1.2757	1.5466	1.0521
1.0000	0.0000	0.0000	0.0000	4.0299	0.3977	0.0000	1.0000	1.2699	1.4566	1.0521
1.0000	0.0000	0.0000	0.0000	4.1666	0.3666	0.0000	1.0000	1.2657	1.3611	1.0521
1.0000	0.0000	0.0000	0.0000	4.3066	0.3344	0.0000	1.0000	1.2633	1.2611	1.0521
1.0000	0.0000	0.0000	0.0000	4.4499	0.3011	0.0000	1.0000	1.2622	1.1566	1.0521
1.0000	0.0000	0.0000	0.0000	4.5966	0.2677	0.0000	1.0000	1.2633	1.0466	1.0521
1.0000	0.0000	0.0000	0.0000	4.7466	0.2333	0.0000	1.0000	1.2677	0.9311	1.0521
1.0000	0.0000	0.0000	0.0000	4.8999	0.1977	0.0000	1.0000	1.2757	0.8111	1.0521
1.0000	0.0000	0.0000	0.0000	5.0566	0.1611	0.0000	1.0000	1.2877	0.6866	1.0521
1.0000	0.0000	0.0000	0.0000	5.2166	0.1244	0.0000	1.0000	1.3033	0.5566	1.0521
1.0000	0.0000	0.0000	0.0000	5.3799	0.0877	0.0000	1.0000	1.3222	0.4211	1.0521
1.0000	0.0000	0.0000	0.0000	5.5466	0.0511	0.0000	1.0000	1.3457	0.2811	1.0521
1.0000	0.0000	0.0000	0.0000	5.7166	0.0144	0.0000	1.0000	1.3733	0.1366	1.0521
1.0000	0.0000	0.0000	0.0000	5.8899	0.0000	0.0000	1.0000	1.4057	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	6.0666	0.0000	0.0000	1.0000	1.4433	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	6.2466	0.0000	0.0000	1.0000	1.4866	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	6.4299	0.0000	0.0000	1.0000	1.5357	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	6.6166	0.0000	0.0000	1.0000	1.5911	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	6.8066	0.0000	0.0000	1.0000	1.6533	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	6.9999	0.0000	0.0000	1.0000	1.7222	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	7.1966	0.0000	0.0000	1.0000	1.7977	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	7.3966	0.0000	0.0000	1.0000	1.8800	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	7.5999	0.0000	0.0000	1.0000	1.9699	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	7.8066	0.0000	0.0000	1.0000	2.0666	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	8.0166	0.0000	0.0000	1.0000	2.1711	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	8.2299	0.0000	0.0000	1.0000	2.2833	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	8.4466	0.0000	0.0000	1.0000	2.4033	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	8.6666	0.0000	0.0000	1.0000	2.5311	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	8.8899	0.0000	0.0000	1.0000	2.6677	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	9.1166	0.0000	0.0000	1.0000	2.8133	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	9.3466	0.0000	0.0000	1.0000	2.9689	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	9.5799	0.0000	0.0000	1.0000	3.1357	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	9.8166	0.0000	0.0000	1.0000	3.3144	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	10.0566	0.0000	0.0000	1.0000	3.5066	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	10.2999	0.0000	0.0000	1.0000	3.7133	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	10.5466	0.0000	0.0000	1.0000	3.9357	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	10.7966	0.0000	0.0000	1.0000	4.1744	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	11.0499	0.0000	0.0000	1.0000	4.4311	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	11.3066	0.0000	0.0000	1.0000	4.7066	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	11.5666	0.0000	0.0000	1.0000	5.0011	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	11.8299	0.0000	0.0000	1.0000	5.3157	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	12.0966	0.0000	0.0000	1.0000	5.6511	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	12.3666	0.0000	0.0000	1.0000	6.0089	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	12.6399	0.0000	0.0000	1.0000	6.3899	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	12.9166	0.0000	0.0000	1.0000	6.7957	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	13.1966	0.0000	0.0000	1.0000	7.2277	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	13.4799	0.0000	0.0000	1.0000	7.6866	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	13.7666	0.0000	0.0000	1.0000	8.1733	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	14.0566	0.0000	0.0000	1.0000	8.6889	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	14.3499	0.0000	0.0000	1.0000	9.2333	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	14.6466	0.0000	0.0000	1.0000	9.8077	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	14.9466	0.0000	0.0000	1.0000	10.4133	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	15.2499	0.0000	0.0000	1.0000	11.0511	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	15.5566	0.0000	0.0000	1.0000	11.7222	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	15.8666	0.0000	0.0000	1.0000	12.4277	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	16.1799	0.0000	0.0000	1.0000	13.1689	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	16.4966	0.0000	0.0000	1.0000	13.9466	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	16.8166	0.0000	0.0000	1.0000	14.7611	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	17.1399	0.0000	0.0000	1.0000	15.6133	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	17.4666	0.0000	0.0000	1.0000	16.5044	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	17.7966	0.0000	0.0000	1.0000	17.4357	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	18.1299	0.0000	0.0000	1.0000	18.4089	0.0000	1.0521
1.0000	0.0000	0.0000	0.0000	18.4666						

TABLE 12A. Lanchester-Clifford-Schlafli Functions  $F_q(x)$ ,  $H_{1-q}(x)$ , and

$T_n(x)$  for  $\alpha = 4/7$  and  $x$  from 0.00 to 1.50.

TABLE 12B. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

 $T_{\alpha}(x)$  for  $\alpha = 4/7$  and  $x$  from 1.50 to 10.0.



Q - 5/7

[illegible]

TABLE 13A. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$T_{\alpha}(x)$  for  $\alpha = 5/7$  and  $x$  from 0.00 to 1.50.



$\alpha = 5/7$ [illegible]

TABLE 13B. Lanchester-Clifford-Schlöfli Functions  $F_\alpha(x)$ ,  $H_{1-\alpha}(x)$ , and  $T_\alpha(x)$  for  $\alpha = 5/7$  and  $x$  from 1.50 to 10.0.

$\alpha = 4/9$

$x$	$F_{4/9}(x)$	$H_{5/9}(x)$	$T_{4/9}(x)$	$x$	$F_{4/9}(x)$	$H_{5/9}(x)$	$T_{4/9}(x)$	$x$	$F_{4/9}(x)$	$H_{5/9}(x)$	$T_{4/9}(x)$
0.0000	1.1111	0.0000	0.0000	1.00	1.1111	0.0000	0.0000	1.00	1.1111	0.0000	0.0000
0.0005	1.1111	0.0000	0.0000	1.01	1.1111	0.0000	0.0000	1.01	1.1111	0.0000	0.0000
0.0010	1.1111	0.0000	0.0000	1.02	1.1111	0.0000	0.0000	1.02	1.1111	0.0000	0.0000
0.0015	1.1111	0.0000	0.0000	1.03	1.1111	0.0000	0.0000	1.03	1.1111	0.0000	0.0000
0.0020	1.1111	0.0000	0.0000	1.04	1.1111	0.0000	0.0000	1.04	1.1111	0.0000	0.0000
0.0025	1.1111	0.0000	0.0000	1.05	1.1111	0.0000	0.0000	1.05	1.1111	0.0000	0.0000
0.0030	1.1111	0.0000	0.0000	1.06	1.1111	0.0000	0.0000	1.06	1.1111	0.0000	0.0000
0.0035	1.1111	0.0000	0.0000	1.07	1.1111	0.0000	0.0000	1.07	1.1111	0.0000	0.0000
0.0040	1.1111	0.0000	0.0000	1.08	1.1111	0.0000	0.0000	1.08	1.1111	0.0000	0.0000
0.0045	1.1111	0.0000	0.0000	1.09	1.1111	0.0000	0.0000	1.09	1.1111	0.0000	0.0000
0.0050	1.1111	0.0000	0.0000	1.10	1.1111	0.0000	0.0000	1.10	1.1111	0.0000	0.0000
0.0055	1.1111	0.0000	0.0000	1.11	1.1111	0.0000	0.0000	1.11	1.1111	0.0000	0.0000
0.0060	1.1111	0.0000	0.0000	1.12	1.1111	0.0000	0.0000	1.12	1.1111	0.0000	0.0000
0.0065	1.1111	0.0000	0.0000	1.13	1.1111	0.0000	0.0000	1.13	1.1111	0.0000	0.0000
0.0070	1.1111	0.0000	0.0000	1.14	1.1111	0.0000	0.0000	1.14	1.1111	0.0000	0.0000
0.0075	1.1111	0.0000	0.0000	1.15	1.1111	0.0000	0.0000	1.15	1.1111	0.0000	0.0000
0.0080	1.1111	0.0000	0.0000	1.16	1.1111	0.0000	0.0000	1.16	1.1111	0.0000	0.0000
0.0085	1.1111	0.0000	0.0000	1.17	1.1111	0.0000	0.0000	1.17	1.1111	0.0000	0.0000
0.0090	1.1111	0.0000	0.0000	1.18	1.1111	0.0000	0.0000	1.18	1.1111	0.0000	0.0000
0.0095	1.1111	0.0000	0.0000	1.19	1.1111	0.0000	0.0000	1.19	1.1111	0.0000	0.0000
0.0100	1.1111	0.0000	0.0000	1.20	1.1111	0.0000	0.0000	1.20	1.1111	0.0000	0.0000
0.0105	1.1111	0.0000	0.0000	1.21	1.1111	0.0000	0.0000	1.21	1.1111	0.0000	0.0000
0.0110	1.1111	0.0000	0.0000	1.22	1.1111	0.0000	0.0000	1.22	1.1111	0.0000	0.0000
0.0115	1.1111	0.0000	0.0000	1.23	1.1111	0.0000	0.0000	1.23	1.1111	0.0000	0.0000
0.0120	1.1111	0.0000	0.0000	1.24	1.1111	0.0000	0.0000	1.24	1.1111	0.0000	0.0000
0.0125	1.1111	0.0000	0.0000	1.25	1.1111	0.0000	0.0000	1.25	1.1111	0.0000	0.0000
0.0130	1.1111	0.0000	0.0000	1.26	1.1111	0.0000	0.0000	1.26	1.1111	0.0000	0.0000
0.0135	1.1111	0.0000	0.0000	1.27	1.1111	0.0000	0.0000	1.27	1.1111	0.0000	0.0000
0.0140	1.1111	0.0000	0.0000	1.28	1.1111	0.0000	0.0000	1.28	1.1111	0.0000	0.0000
0.0145	1.1111	0.0000	0.0000	1.29	1.1111	0.0000	0.0000	1.29	1.1111	0.0000	0.0000
0.0150	1.1111	0.0000	0.0000	1.30	1.1111	0.0000	0.0000	1.30	1.1111	0.0000	0.0000
0.0155	1.1111	0.0000	0.0000	1.31	1.1111	0.0000	0.0000	1.31	1.1111	0.0000	0.0000
0.0160	1.1111	0.0000	0.0000	1.32	1.1111	0.0000	0.0000	1.32	1.1111	0.0000	0.0000
0.0165	1.1111	0.0000	0.0000	1.33	1.1111	0.0000	0.0000	1.33	1.1111	0.0000	0.0000
0.0170	1.1111	0.0000	0.0000	1.34	1.1111	0.0000	0.0000	1.34	1.1111	0.0000	0.0000
0.0175	1.1111	0.0000	0.0000	1.35	1.1111	0.0000	0.0000	1.35	1.1111	0.0000	0.0000
0.0180	1.1111	0.0000	0.0000	1.36	1.1111	0.0000	0.0000	1.36	1.1111	0.0000	0.0000
0.0185	1.1111	0.0000	0.0000	1.37	1.1111	0.0000	0.0000	1.37	1.1111	0.0000	0.0000
0.0190	1.1111	0.0000	0.0000	1.38	1.1111	0.0000	0.0000	1.38	1.1111	0.0000	0.0000
0.0195	1.1111	0.0000	0.0000	1.39	1.1111	0.0000	0.0000	1.39	1.1111	0.0000	0.0000
0.0200	1.1111	0.0000	0.0000	1.40	1.1111	0.0000	0.0000	1.40	1.1111	0.0000	0.0000
0.0205	1.1111	0.0000	0.0000	1.41	1.1111	0.0000	0.0000	1.41	1.1111	0.0000	0.0000
0.0210	1.1111	0.0000	0.0000	1.42	1.1111	0.0000	0.0000	1.42	1.1111	0.0000	0.0000
0.0215	1.1111	0.0000	0.0000	1.43	1.1111	0.0000	0.0000	1.43	1.1111	0.0000	0.0000
0.0220	1.1111	0.0000	0.0000	1.44	1.1111	0.0000	0.0000	1.44	1.1111	0.0000	0.0000
0.0225	1.1111	0.0000	0.0000	1.45	1.1111	0.0000	0.0000	1.45	1.1111	0.0000	0.0000
0.0230	1.1111	0.0000	0.0000	1.46	1.1111	0.0000	0.0000	1.46	1.1111	0.0000	0.0000
0.0235	1.1111	0.0000	0.0000	1.47	1.1111	0.0000	0.0000	1.47	1.1111	0.0000	0.0000
0.0240	1.1111	0.0000	0.0000	1.48	1.1111	0.0000	0.0000	1.48	1.1111	0.0000	0.0000
0.0245	1.1111	0.0000	0.0000	1.49	1.1111	0.0000	0.0000	1.49	1.1111	0.0000	0.0000
0.0250	1.1111	0.0000	0.0000	1.50	1.1111	0.0000	0.0000	1.50	1.1111	0.0000	0.0000

TABLE 14A. Lanchester-Clifford-Schlafli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$T_{\alpha}(x)$  for  $\alpha = 4/9$  and  $x$  from 0.00 to 1.50.



[illegible]

TABLE 14B. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$$T_\alpha(x) \text{ for } \alpha = 4/9 \text{ and } x \text{ from } 1.50 \text{ to } 10.0.$$









[illegible]

TABLE 16A. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$T_\alpha(x)$  for  $\alpha = 3/11$  and  $x$  from 0.00 to 1.50.





$\alpha = 5/11$

$x$	$F_{5/11}(x)$	$H_{6/11}(x)$	$T_{5/11}(x)$	$x$	$F_{5/11}(x)$	$H_{6/11}(x)$	$T_{5/11}(x)$	$x$	$F_{5/11}(x)$	$H_{6/11}(x)$	$T_{5/11}(x)$
0.01	1.00000	0.00000	0.00000	0.51	1.14221	0.47041	0.39880	1.01	1.59990	1.00992	0.61974
0.02	1.00002	0.00002	0.00002	0.52	1.15221	0.47042	0.39881	1.02	1.61198	1.02103	0.63340
0.03	1.00008	0.00008	0.00008	0.53	1.16224	0.47048	0.39887	1.03	1.62425	1.03253	0.64699
0.04	1.00018	0.00018	0.00018	0.54	1.17229	0.47059	0.39897	1.04	1.63673	1.04439	0.66052
0.05	1.00030	0.00030	0.00030	0.55	1.18243	0.47104	0.40193	1.05	1.64941	1.05662	0.67400
0.06	1.00046	0.00046	0.00046	0.56	1.19275	0.47175	0.41116	1.06	1.66231	1.06929	0.68738
0.07	1.00067	0.00067	0.00067	0.57	1.20320	0.47272	0.42144	1.07	1.67543	1.08235	0.70069
0.08	1.00093	0.00093	0.00093	0.58	1.21380	0.47390	0.43280	1.08	1.68877	1.09579	0.71391
0.09	1.00124	0.00124	0.00124	0.59	1.22454	0.47531	0.44519	1.09	1.70233	1.10962	0.72705
0.10	1.00161	0.00161	0.00161	0.60	1.23542	0.47694	0.45869	1.10	1.71613	1.12383	0.74012
0.11	1.00204	0.00204	0.00204	0.61	1.24653	0.47880	0.47326	1.11	1.73017	1.13843	0.75312
0.12	1.00253	0.00253	0.00253	0.62	1.25787	0.48089	0.48897	1.12	1.74445	1.15343	0.76604
0.13	1.00308	0.00308	0.00308	0.63	1.26944	0.48322	0.50589	1.13	1.75897	1.16883	0.77887
0.14	1.00369	0.00369	0.00369	0.64	1.28124	0.48579	0.52405	1.14	1.77373	1.18463	0.79161
0.15	1.00436	0.00436	0.00436	0.65	1.29327	0.48859	0.54349	1.15	1.78873	1.20083	0.80428
0.16	1.00509	0.00509	0.00509	0.66	1.30553	0.49162	0.56426	1.16	1.80397	1.21743	0.81688
0.17	1.00588	0.00588	0.00588	0.67	1.31803	0.49490	0.58644	1.17	1.81945	1.23443	0.82935
0.18	1.00673	0.00673	0.00673	0.68	1.33077	0.49843	0.61005	1.18	1.83517	1.25183	0.84168
0.19	1.00764	0.00764	0.00764	0.69	1.34375	0.50222	0.63519	1.19	1.85113	1.26963	0.85388
0.20	1.00861	0.00861	0.00861	0.70	1.35697	0.50627	0.66189	1.20	1.86733	1.28783	0.86595
0.21	1.00964	0.00964	0.00964	0.71	1.37043	0.51059	0.68917	1.21	1.88377	1.30643	0.87788
0.22	1.01073	0.01073	0.01073	0.72	1.38413	0.51519	0.71694	1.22	1.90045	1.32543	0.88968
0.23	1.01188	0.01188	0.01188	0.73	1.39807	0.52007	0.74529	1.23	1.91737	1.34483	0.90135
0.24	1.01309	0.01309	0.01309	0.74	1.41225	0.52523	0.77423	1.24	1.93453	1.36463	0.91288
0.25	1.01436	0.01436	0.01436	0.75	1.42657	0.53067	0.80377	1.25	1.95193	1.38483	0.92428
0.26	1.01569	0.01569	0.01569	0.76	1.44113	0.53637	0.83391	1.26	1.96957	1.40543	0.93555
0.27	1.01708	0.01708	0.01708	0.77	1.45593	0.54233	0.86465	1.27	1.98745	1.42643	0.94668
0.28	1.01853	0.01853	0.01853	0.78	1.47097	0.54855	0.89601	1.28	2.00557	1.44783	0.95768
0.29	1.02004	0.02004	0.02004	0.79	1.48625	0.55503	0.92809	1.29	2.02393	1.46963	0.96855
0.30	1.02161	0.02161	0.02161	0.80	1.50177	0.56177	0.96081	1.30	2.04253	1.49183	0.97928
0.31	1.02324	0.02324	0.02324	0.81	1.51753	0.56877	0.99417	1.31	2.06137	1.51443	0.99000
0.32	1.02493	0.02493	0.02493	0.82	1.53353	0.57603	1.02819	1.32	2.08045	1.53743	1.00075
0.33	1.02668	0.02668	0.02668	0.83	1.54977	0.58355	1.06287	1.33	2.10000	1.56083	1.01150
0.34	1.02849	0.02849	0.02849	0.84	1.56625	0.59133	1.09821	1.34	2.12000	1.58463	1.02225
0.35	1.03036	0.03036	0.03036	0.85	1.58297	0.59937	1.13431	1.35	2.14050	1.60883	1.03300
0.36	1.03229	0.03229	0.03229	0.86	1.60000	0.60767	1.17117	1.36	2.16150	1.63343	1.04375
0.37	1.03428	0.03428	0.03428	0.87	1.61725	0.61623	1.20879	1.37	2.18300	1.65843	1.05450
0.38	1.03633	0.03633	0.03633	0.88	1.63473	0.62505	1.24717	1.38	2.20500	1.68383	1.06525
0.39	1.03844	0.03844	0.03844	0.89	1.65243	0.63413	1.28631	1.39	2.22750	1.70963	1.07600
0.40	1.04061	0.04061	0.04061	0.90	1.67035	0.64347	1.32621	1.40	2.25050	1.73583	1.08675
0.41	1.04284	0.04284	0.04284	0.91	1.68849	0.65307	1.36687	1.41	2.27400	1.76243	1.09750
0.42	1.04513	0.04513	0.04513	0.92	1.70685	0.66293	1.40829	1.42	2.29800	1.78943	1.10825
0.43	1.04748	0.04748	0.04748	0.93	1.72543	0.67305	1.45049	1.43	2.32250	1.81683	1.11900
0.44	1.04989	0.04989	0.04989	0.94	1.74423	0.68343	1.49349	1.44	2.34750	1.84463	1.12975
0.45	1.05236	0.05236	0.05236	0.95	1.76325	0.69407	1.53729	1.45	2.37300	1.87283	1.14050
0.46	1.05489	0.05489	0.05489	0.96	1.78249	0.70497	1.58189	1.46	2.40000	1.90143	1.15125
0.47	1.05748	0.05748	0.05748	0.97	1.80195	0.71613	1.62729	1.47	2.42750	1.93043	1.16200
0.48	1.06013	0.06013	0.06013	0.98	1.82163	0.72755	1.67349	1.48	2.45550	1.95983	1.17275
0.49	1.06284	0.06284	0.06284	0.99	1.84153	0.73923	1.72049	1.49	2.48400	1.98963	1.18350
0.50	1.06561	0.06561	0.06561	1.00	1.86165	0.75117	1.76829	1.50	2.51300	2.01983	1.19425

TABLE 17A. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$T_{\alpha}(x)$  for  $\alpha = 5/11$  and  $x$  from 0.00 to 1.50.



$\alpha = 5/11$ [illegible]

TABLE 17B. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$$T_{\alpha}(x) \text{ for } \alpha = 5/11 \text{ and } x \text{ from } 1.50 \text{ to } 10.0.$$











$\alpha = 8/11$

$x$	$F_{8/11}(x)$	$H_{3/11}(x)$	$T_{8/11}(x)$	$x$	$F_{8/11}(x)$	$H_{3/11}(x)$	$T_{8/11}(x)$	$x$	$F_{8/11}(x)$	$H_{3/11}(x)$	$T_{8/11}(x)$
0.0000	1.00000	1.00000	2.00000	0.1000	0.99999	0.99999	1.99999	0.2000	0.99996	0.99996	1.99996
0.1000	0.99999	0.99999	1.99999	0.2000	0.99996	0.99996	1.99996	0.3000	0.99990	0.99990	1.99990
0.2000	0.99996	0.99996	1.99996	0.3000	0.99990	0.99990	1.99990	0.4000	0.99980	0.99980	1.99980
0.3000	0.99980	0.99980	1.99980	0.4000	0.99980	0.99980	1.99980	0.5000	0.99965	0.99965	1.99965
0.4000	0.99965	0.99965	1.99965	0.5000	0.99965	0.99965	1.99965	0.6000	0.99945	0.99945	1.99945
0.5000	0.99945	0.99945	1.99945	0.6000	0.99945	0.99945	1.99945	0.7000	0.99920	0.99920	1.99920
0.6000	0.99920	0.99920	1.99920	0.7000	0.99920	0.99920	1.99920	0.8000	0.99890	0.99890	1.99890
0.7000	0.99890	0.99890	1.99890	0.8000	0.99890	0.99890	1.99890	0.9000	0.99855	0.99855	1.99855
0.8000	0.99855	0.99855	1.99855	0.9000	0.99855	0.99855	1.99855	1.0000	0.99815	0.99815	1.99815
0.9000	0.99815	0.99815	1.99815	1.0000	0.99815	0.99815	1.99815				

TABLE 19B. Lanchester-Clifford-Schlafli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$T_{\alpha}(x)$  for  $\alpha = 8/11$  and  $x$  from 1.50 to 10.0.







[illegible]

TABLE 21A. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

 $T_{\alpha}(x)$  for  $\alpha = 8/13$  and  $x$  from 0.00 to 1.50.



$\alpha = 8/13$

$T_8/13(x)$

$H_5/13(x)$

$F_8/13(x)$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x$

$x</$

$\alpha = 5/17$

$x$	$F_{5/17}(x)$	$H_{12/17}(x)$	$T_{5/17}(x)$	$x$	$F_{5/17}(x)$	$H_{12/17}(x)$	$T_{5/17}(x)$	$x$	$F_{5/17}(x)$	$H_{12/17}(x)$	$T_{5/17}(x)$
0.0000	1.00000	0.00000	0.00000	1.0000	1.1668	0.2754	0.1744	1.0000	0.9514	0.4148	0.3173
0.0001	1.00004	0.00000	0.00000	0.9999	1.1667	0.2754	0.1744	1.0001	0.9514	0.4148	0.3173
0.0002	1.00008	0.00000	0.00000	0.9998	1.1666	0.2754	0.1744	1.0002	0.9514	0.4148	0.3173
0.0003	1.00012	0.00000	0.00000	0.9997	1.1665	0.2754	0.1744	1.0003	0.9514	0.4148	0.3173
0.0004	1.00016	0.00000	0.00000	0.9996	1.1664	0.2754	0.1744	1.0004	0.9514	0.4148	0.3173
0.0005	1.00020	0.00000	0.00000	0.9995	1.1663	0.2754	0.1744	1.0005	0.9514	0.4148	0.3173
0.0006	1.00024	0.00000	0.00000	0.9994	1.1662	0.2754	0.1744	1.0006	0.9514	0.4148	0.3173
0.0007	1.00028	0.00000	0.00000	0.9993	1.1661	0.2754	0.1744	1.0007	0.9514	0.4148	0.3173
0.0008	1.00032	0.00000	0.00000	0.9992	1.1660	0.2754	0.1744	1.0008	0.9514	0.4148	0.3173
0.0009	1.00036	0.00000	0.00000	0.9991	1.1659	0.2754	0.1744	1.0009	0.9514	0.4148	0.3173
0.0010	1.00040	0.00000	0.00000	0.9990	1.1658	0.2754	0.1744	1.0010	0.9514	0.4148	0.3173
0.0011	1.00044	0.00000	0.00000	0.9989	1.1657	0.2754	0.1744	1.0011	0.9514	0.4148	0.3173
0.0012	1.00048	0.00000	0.00000	0.9988	1.1656	0.2754	0.1744	1.0012	0.9514	0.4148	0.3173
0.0013	1.00052	0.00000	0.00000	0.9987	1.1655	0.2754	0.1744	1.0013	0.9514	0.4148	0.3173
0.0014	1.00056	0.00000	0.00000	0.9986	1.1654	0.2754	0.1744	1.0014	0.9514	0.4148	0.3173
0.0015	1.00060	0.00000	0.00000	0.9985	1.1653	0.2754	0.1744	1.0015	0.9514	0.4148	0.3173
0.0016	1.00064	0.00000	0.00000	0.9984	1.1652	0.2754	0.1744	1.0016	0.9514	0.4148	0.3173
0.0017	1.00068	0.00000	0.00000	0.9983	1.1651	0.2754	0.1744	1.0017	0.9514	0.4148	0.3173
0.0018	1.00072	0.00000	0.00000	0.9982	1.1650	0.2754	0.1744	1.0018	0.9514	0.4148	0.3173
0.0019	1.00076	0.00000	0.00000	0.9981	1.1649	0.2754	0.1744	1.0019	0.9514	0.4148	0.3173
0.0020	1.00080	0.00000	0.00000	0.9980	1.1648	0.2754	0.1744	1.0020	0.9514	0.4148	0.3173
0.0021	1.00084	0.00000	0.00000	0.9979	1.1647	0.2754	0.1744	1.0021	0.9514	0.4148	0.3173
0.0022	1.00088	0.00000	0.00000	0.9978	1.1646	0.2754	0.1744	1.0022	0.9514	0.4148	0.3173
0.0023	1.00092	0.00000	0.00000	0.9977	1.1645	0.2754	0.1744	1.0023	0.9514	0.4148	0.3173
0.0024	1.00096	0.00000	0.00000	0.9976	1.1644	0.2754	0.1744	1.0024	0.9514	0.4148	0.3173
0.0025	1.00100	0.00000	0.00000	0.9975	1.1643	0.2754	0.1744	1.0025	0.9514	0.4148	0.3173
0.0026	1.00104	0.00000	0.00000	0.9974	1.1642	0.2754	0.1744	1.0026	0.9514	0.4148	0.3173
0.0027	1.00108	0.00000	0.00000	0.9973	1.1641	0.2754	0.1744	1.0027	0.9514	0.4148	0.3173
0.0028	1.00112	0.00000	0.00000	0.9972	1.1640	0.2754	0.1744	1.0028	0.9514	0.4148	0.3173
0.0029	1.00116	0.00000	0.00000	0.9971	1.1639	0.2754	0.1744	1.0029	0.9514	0.4148	0.3173
0.0030	1.00120	0.00000	0.00000	0.9970	1.1638	0.2754	0.1744	1.0030	0.9514	0.4148	0.3173
0.0031	1.00124	0.00000	0.00000	0.9969	1.1637	0.2754	0.1744	1.0031	0.9514	0.4148	0.3173
0.0032	1.00128	0.00000	0.00000	0.9968	1.1636	0.2754	0.1744	1.0032	0.9514	0.4148	0.3173
0.0033	1.00132	0.00000	0.00000	0.9967	1.1635	0.2754	0.1744	1.0033	0.9514	0.4148	0.3173
0.0034	1.00136	0.00000	0.00000	0.9966	1.1634	0.2754	0.1744	1.0034	0.9514	0.4148	0.3173
0.0035	1.00140	0.00000	0.00000	0.9965	1.1633	0.2754	0.1744	1.0035	0.9514	0.4148	0.3173
0.0036	1.00144	0.00000	0.00000	0.9964	1.1632	0.2754	0.1744	1.0036	0.9514	0.4148	0.3173
0.0037	1.00148	0.00000	0.00000	0.9963	1.1631	0.2754	0.1744	1.0037	0.9514	0.4148	0.3173
0.0038	1.00152	0.00000	0.00000	0.9962	1.1630	0.2754	0.1744	1.0038	0.9514	0.4148	0.3173
0.0039	1.00156	0.00000	0.00000	0.9961	1.1629	0.2754	0.1744	1.0039	0.9514	0.4148	0.3173
0.0040	1.00160	0.00000	0.00000	0.9960	1.1628	0.2754	0.1744	1.0040	0.9514	0.4148	0.3173
0.0041	1.00164	0.00000	0.00000	0.9959	1.1627	0.2754	0.1744	1.0041	0.9514	0.4148	0.3173
0.0042	1.00168	0.00000	0.00000	0.9958	1.1626	0.2754	0.1744	1.0042	0.9514	0.4148	0.3173
0.0043	1.00172	0.00000	0.00000	0.9957	1.1625	0.2754	0.1744	1.0043	0.9514	0.4148	0.3173
0.0044	1.00176	0.00000	0.00000	0.9956	1.1624	0.2754	0.1744	1.0044	0.9514	0.4148	0.3173
0.0045	1.00180	0.00000	0.00000	0.9955	1.1623	0.2754	0.1744	1.0045	0.9514	0.4148	0.3173
0.0046	1.00184	0.00000	0.00000	0.9954	1.1622	0.2754	0.1744	1.0046	0.9514	0.4148	0.3173
0.0047	1.00188	0.00000	0.00000	0.9953	1.1621	0.2754	0.1744	1.0047	0.9514	0.4148	0.3173
0.0048	1.00192	0.00000	0.00000	0.9952	1.1620	0.2754	0.1744	1.0048	0.9514	0.4148	0.3173
0.0049	1.00196	0.00000	0.00000	0.9951	1.1619	0.2754	0.1744	1.0049	0.9514	0.4148	0.3173
0.0050	1.00200	0.00000	0.00000	0.9950	1.1618	0.2754	0.1744	1.0050	0.9514	0.4148	0.3173

TABLE 22A. Lanchester-Clifford-Schlafli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$T_{\alpha}(x)$  for  $\alpha = 5/17$  and  $x$  from 0.00 to 1.50.



$\alpha = 5/17$

$x$	$F_{5/17}(x)$	$H_{12/17}(x)$	$T_{5/17}(x)$	$x$	$F_{5/17}(x)$	$H_{12/17}(x)$	$T_{5/17}(x)$	$x$	$F_{5/17}(x)$	$H_{12/17}(x)$	$T_{5/17}(x)$
0.00000	3.36340	1.89955	0.38324	0.00000	5.91999	3.11144	0.40788	6.0	426.03496	179.75802	0.42195
0.00001	3.36340	1.89955	0.38324	0.00001	5.91999	3.11144	0.40788	6.1	426.03496	179.75802	0.42195
0.00002	3.36340	1.89955	0.38324	0.00002	5.91999	3.11144	0.40788	6.2	426.03496	179.75802	0.42195
0.00003	3.36340	1.89955	0.38324	0.00003	5.91999	3.11144	0.40788	6.3	426.03496	179.75802	0.42195
0.00004	3.36340	1.89955	0.38324	0.00004	5.91999	3.11144	0.40788	6.4	426.03496	179.75802	0.42195
0.00005	3.36340	1.89955	0.38324	0.00005	5.91999	3.11144	0.40788	6.5	426.03496	179.75802	0.42195
0.00006	3.36340	1.89955	0.38324	0.00006	5.91999	3.11144	0.40788	6.6	426.03496	179.75802	0.42195
0.00007	3.36340	1.89955	0.38324	0.00007	5.91999	3.11144	0.40788	6.7	426.03496	179.75802	0.42195
0.00008	3.36340	1.89955	0.38324	0.00008	5.91999	3.11144	0.40788	6.8	426.03496	179.75802	0.42195
0.00009	3.36340	1.89955	0.38324	0.00009	5.91999	3.11144	0.40788	6.9	426.03496	179.75802	0.42195
0.00010	3.36340	1.89955	0.38324	0.00010	5.91999	3.11144	0.40788	7.0	426.03496	179.75802	0.42195
0.00011	3.36340	1.89955	0.38324	0.00011	5.91999	3.11144	0.40788	7.1	426.03496	179.75802	0.42195
0.00012	3.36340	1.89955	0.38324	0.00012	5.91999	3.11144	0.40788	7.2	426.03496	179.75802	0.42195
0.00013	3.36340	1.89955	0.38324	0.00013	5.91999	3.11144	0.40788	7.3	426.03496	179.75802	0.42195
0.00014	3.36340	1.89955	0.38324	0.00014	5.91999	3.11144	0.40788	7.4	426.03496	179.75802	0.42195
0.00015	3.36340	1.89955	0.38324	0.00015	5.91999	3.11144	0.40788	7.5	426.03496	179.75802	0.42195
0.00016	3.36340	1.89955	0.38324	0.00016	5.91999	3.11144	0.40788	7.6	426.03496	179.75802	0.42195
0.00017	3.36340	1.89955	0.38324	0.00017	5.91999	3.11144	0.40788	7.7	426.03496	179.75802	0.42195
0.00018	3.36340	1.89955	0.38324	0.00018	5.91999	3.11144	0.40788	7.8	426.03496	179.75802	0.42195
0.00019	3.36340	1.89955	0.38324	0.00019	5.91999	3.11144	0.40788	7.9	426.03496	179.75802	0.42195
0.00020	3.36340	1.89955	0.38324	0.00020	5.91999	3.11144	0.40788	8.0	426.03496	179.75802	0.42195
0.00021	3.36340	1.89955	0.38324	0.00021	5.91999	3.11144	0.40788	8.1	426.03496	179.75802	0.42195
0.00022	3.36340	1.89955	0.38324	0.00022	5.91999	3.11144	0.40788	8.2	426.03496	179.75802	0.42195
0.00023	3.36340	1.89955	0.38324	0.00023	5.91999	3.11144	0.40788	8.3	426.03496	179.75802	0.42195
0.00024	3.36340	1.89955	0.38324	0.00024	5.91999	3.11144	0.40788	8.4	426.03496	179.75802	0.42195
0.00025	3.36340	1.89955	0.38324	0.00025	5.91999	3.11144	0.40788	8.5	426.03496	179.75802	0.42195
0.00026	3.36340	1.89955	0.38324	0.00026	5.91999	3.11144	0.40788	8.6	426.03496	179.75802	0.42195
0.00027	3.36340	1.89955	0.38324	0.00027	5.91999	3.11144	0.40788	8.7	426.03496	179.75802	0.42195
0.00028	3.36340	1.89955	0.38324	0.00028	5.91999	3.11144	0.40788	8.8	426.03496	179.75802	0.42195
0.00029	3.36340	1.89955	0.38324	0.00029	5.91999	3.11144	0.40788	8.9	426.03496	179.75802	0.42195
0.00030	3.36340	1.89955	0.38324	0.00030	5.91999	3.11144	0.40788	9.0	426.03496	179.75802	0.42195
0.00031	3.36340	1.89955	0.38324	0.00031	5.91999	3.11144	0.40788	9.1	426.03496	179.75802	0.42195
0.00032	3.36340	1.89955	0.38324	0.00032	5.91999	3.11144	0.40788	9.2	426.03496	179.75802	0.42195
0.00033	3.36340	1.89955	0.38324	0.00033	5.91999	3.11144	0.40788	9.3	426.03496	179.75802	0.42195
0.00034	3.36340	1.89955	0.38324	0.00034	5.91999	3.11144	0.40788	9.4	426.03496	179.75802	0.42195
0.00035	3.36340	1.89955	0.38324	0.00035	5.91999	3.11144	0.40788	9.5	426.03496	179.75802	0.42195
0.00036	3.36340	1.89955	0.38324	0.00036	5.91999	3.11144	0.40788	9.6	426.03496	179.75802	0.42195
0.00037	3.36340	1.89955	0.38324	0.00037	5.91999	3.11144	0.40788	9.7	426.03496	179.75802	0.42195
0.00038	3.36340	1.89955	0.38324	0.00038	5.91999	3.11144	0.40788	9.8	426.03496	179.75802	0.42195
0.00039	3.36340	1.89955	0.38324	0.00039	5.91999	3.11144	0.40788	9.9	426.03496	179.75802	0.42195
0.00040	3.36340	1.89955	0.38324	0.00040	5.91999	3.11144	0.40788	10.0	426.03496	179.75802	0.42195

TABLE 22B. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and

$T_{\alpha}(x)$  for  $\alpha = 5/17$  and  $x$  from 1.50 to 10.0.

[illegible]

Lanchester-Clifford-Schlöfli Functions  $F_\alpha(x)$ ,  $H_{1-\alpha}(x)$ , and  $T_\alpha(x)$  for  $\alpha = 12/17$  and  $x$  from 0.00 to 1.50.





[illegible]

TABLE 24A. Lanchester-Clifford-Schlöfli Functions  $F_{\alpha}(x)$ ,  $H_{1-q}(x)$ , and

$$T_\alpha(x) \text{ for } \alpha = 5/21 \text{ and } x \text{ from } 0.00 \text{ to } 1.50.$$



$\alpha = 5/21$

$x$	$F_{5/21}(x)$	$H_{16/21}(x)$	$T_{5/21}(x)$	$x$	$F_{5/21}(x)$	$H_{16/21}(x)$	$T_{5/21}(x)$	$x$	$F_{5/21}(x)$	$H_{16/21}(x)$	$T_{5/21}(x)$
1.50	3.9441	1.1492	0.2939	2.0	7.1623	2.2049	0.3035	6.0	5.6196	1.8128	0.3169
1.51	3.9444	1.1494	0.2940	2.1	7.1624	2.2050	0.3036	6.1	5.6197	1.8129	0.3170
1.52	3.9447	1.1496	0.2941	2.2	7.1625	2.2051	0.3037	6.2	5.6198	1.8130	0.3171
1.53	3.9450	1.1498	0.2942	2.3	7.1626	2.2052	0.3038	6.3	5.6199	1.8131	0.3172
1.54	3.9453	1.1500	0.2943	2.4	7.1627	2.2053	0.3039	6.4	5.6200	1.8132	0.3173
1.55	3.9456	1.1502	0.2944	2.5	7.1628	2.2054	0.3040	6.5	5.6201	1.8133	0.3174
1.56	3.9459	1.1504	0.2945	2.6	7.1629	2.2055	0.3041	6.6	5.6202	1.8134	0.3175
1.57	3.9462	1.1506	0.2946	2.7	7.1630	2.2056	0.3042	6.7	5.6203	1.8135	0.3176
1.58	3.9465	1.1508	0.2947	2.8	7.1631	2.2057	0.3043	6.8	5.6204	1.8136	0.3177
1.59	3.9468	1.1510	0.2948	2.9	7.1632	2.2058	0.3044	6.9	5.6205	1.8137	0.3178
1.60	3.9471	1.1512	0.2949	3.0	7.1633	2.2059	0.3045	7.0	5.6206	1.8138	0.3179
1.61	3.9474	1.1514	0.2950	3.1	7.1634	2.2060	0.3046	7.1	5.6207	1.8139	0.3180
1.62	3.9477	1.1516	0.2951	3.2	7.1635	2.2061	0.3047	7.2	5.6208	1.8140	0.3181
1.63	3.9480	1.1518	0.2952	3.3	7.1636	2.2062	0.3048	7.3	5.6209	1.8141	0.3182
1.64	3.9483	1.1520	0.2953	3.4	7.1637	2.2063	0.3049	7.4	5.6210	1.8142	0.3183
1.65	3.9486	1.1522	0.2954	3.5	7.1638	2.2064	0.3050	7.5	5.6211	1.8143	0.3184
1.66	3.9489	1.1524	0.2955	3.6	7.1639	2.2065	0.3051	7.6	5.6212	1.8144	0.3185
1.67	3.9492	1.1526	0.2956	3.7	7.1640	2.2066	0.3052	7.7	5.6213	1.8145	0.3186
1.68	3.9495	1.1528	0.2957	3.8	7.1641	2.2067	0.3053	7.8	5.6214	1.8146	0.3187
1.69	3.9498	1.1530	0.2958	3.9	7.1642	2.2068	0.3054	7.9	5.6215	1.8147	0.3188
1.70	3.9501	1.1532	0.2959	4.0	7.1643	2.2069	0.3055	8.0	5.6216	1.8148	0.3189
1.71	3.9504	1.1534	0.2960	4.1	7.1644	2.2070	0.3056	8.1	5.6217	1.8149	0.3190
1.72	3.9507	1.1536	0.2961	4.2	7.1645	2.2071	0.3057	8.2	5.6218	1.8150	0.3191
1.73	3.9510	1.1538	0.2962	4.3	7.1646	2.2072	0.3058	8.3	5.6219	1.8151	0.3192
1.74	3.9513	1.1540	0.2963	4.4	7.1647	2.2073	0.3059	8.4	5.6220	1.8152	0.3193
1.75	3.9516	1.1542	0.2964	4.5	7.1648	2.2074	0.3060	8.5	5.6221	1.8153	0.3194
1.76	3.9519	1.1544	0.2965	4.6	7.1649	2.2075	0.3061	8.6	5.6222	1.8154	0.3195
1.77	3.9522	1.1546	0.2966	4.7	7.1650	2.2076	0.3062	8.7	5.6223	1.8155	0.3196
1.78	3.9525	1.1548	0.2967	4.8	7.1651	2.2077	0.3063	8.8	5.6224	1.8156	0.3197
1.79	3.9528	1.1550	0.2968	4.9	7.1652	2.2078	0.3064	8.9	5.6225	1.8157	0.3198
1.80	3.9531	1.1552	0.2969	5.0	7.1653	2.2079	0.3065	9.0	5.6226	1.8158	0.3199
1.81	3.9534	1.1554	0.2970	5.1	7.1654	2.2080	0.3066	9.1	5.6227	1.8159	0.3200
1.82	3.9537	1.1556	0.2971	5.2	7.1655	2.2081	0.3067	9.2	5.6228	1.8160	0.3201
1.83	3.9540	1.1558	0.2972	5.3	7.1656	2.2082	0.3068	9.3	5.6229	1.8161	0.3202
1.84	3.9543	1.1560	0.2973	5.4	7.1657	2.2083	0.3069	9.4	5.6230	1.8162	0.3203
1.85	3.9546	1.1562	0.2974	5.5	7.1658	2.2084	0.3070	9.5	5.6231	1.8163	0.3204
1.86	3.9549	1.1564	0.2975	5.6	7.1659	2.2085	0.3071	9.6	5.6232	1.8164	0.3205
1.87	3.9552	1.1566	0.2976	5.7	7.1660	2.2086	0.3072	9.7	5.6233	1.8165	0.3206
1.88	3.9555	1.1568	0.2977	5.8	7.1661	2.2087	0.3073	9.8	5.6234	1.8166	0.3207
1.89	3.9558	1.1570	0.2978	5.9	7.1662	2.2088	0.3074	9.9	5.6235	1.8167	0.3208
1.90	3.9561	1.1572	0.2979	6.0	7.1663	2.2089	0.3075	10.0	5.6236	1.8168	0.3209
1.91	3.9564	1.1574	0.2980								
1.92	3.9567	1.1576	0.2981								
1.93	3.9570	1.1578	0.2982								
1.94	3.9573	1.1580	0.2983								
1.95	3.9576	1.1582	0.2984								
1.96	3.9579	1.1584	0.2985								
1.97	3.9582	1.1586	0.2986								
1.98	3.9585	1.1588	0.2987								
1.99	3.9588	1.1590	0.2988								
2.00	3.9591	1.1592	0.2989								

TABLE 24B. Lanchester-Clifford-Schlafli Functions  $F_{\alpha}(x)$ ,  $H_{1-\alpha}(x)$ , and  $T_{\alpha}(x)$  for  $\alpha = 5/21$  and  $x$  from 1.50 to 10.0.

$T_\alpha(x)$  for  $\alpha = 16/21$  and  $x$  from 0.00 to 1.50.

[illegible]



